Global Journal of Current Research Vol. 2 No.3. 2014. Pp.59-63

Full Length Research Paper

Duality of Generalised Semilocally B-Preinvex Function

O. Chandra Sekhara Reddy[1], G. Varalakshmi[2] and P.R.S Reddy[3]

[1]*Associate Professor, Department of Statistics, Ambo University, Ethiopia*
[2] *Head of the Department of Statistics, D.K.G.D. College for Women, Nellore*
[3]*Professor in Department of Statistics, S.V. University, Tirupati*

**Corresponding Author: O. Chandra Sekhara Reddy[1]*

Abstract
In this paper, we derive necessary and sufficient conditions for semi differentiable b-preinvex functions and also weak and strong duality results have been proved.

Keywords: *Semi locally b - preinvex, semi locally Pseudo b - preinvex , semi locally strongly pseudo b - preinvex functions, and Duality*

Introduction

A non linear programming problem is considered where the functions involved are η - semi differentiable. Introduced Non - convex and semi differentiable functions(Kaul, R; N. and Lyall, V) . Introduced preinvex functions in multiple objective optimization(Weir T and Mond, B). Duality in nonlinear programming involving semilocally B-vex and related functions (Suneja, S.K. and Gupta, Sudha) . Introduced generalization of preinvex and B-vex function (Suneja, S.K., Singh C and Bector CR). Introduced Duality in multiple objective programming involving semilocally pre-invex and related functions(Preda v and Stancu - Minasian, IM). Nonlinear programming with semilocally B - preinvex and related functions (Ioan M. Stancu - Minasian). But they have not considered this under the concept of convex. Hence this chapter is made to fill the gap in the aim of research we extend this under the concept of convex Duality of generalized semilocally B - preinvex function.

Definitions

1.1 Definition: Let R^n be n - dimensional Euclidean space and R^n_+ its positive orthant, that is $R^n_+ = \{x \in R^n,\ x = (x_j), x_j \geq 0,\ j = 1,2,\cdots,n\}$. Let $X^* \subseteq R^n$ be a set and X* is convex at if $\bar{x} + \lambda(x - \bar{x}) \in X^*$ for all $x \in X^*$ and $\lambda \in [0,1]$. We say that X° is convex if X* is convex at any $x \in X^*$.

1.2 Definition: Let $X^* \subseteq R^n$ be a non empty set. A function $f : X^* \to R$ is said to be preinvex on X* if there exists an n - dimensional vector function. Such that for all x, $u \in X^*$ and $\lambda \in [0,1]$ we have $f(u + \lambda(x - u)) \leq \lambda f(x) + (1 - \lambda) f(u)$.

1.3 Definition: We say that $X^* \subseteq R^n$ is a locally star shaped set at $\bar{x}\ (\bar{x} \in X^*)$ if, for any $x \in X^*$ exists $0 < a(x, \bar{x}) \leq 1$ such that $\bar{x} + \lambda(x - \bar{x}) \in X^*$ for any $\lambda \in [0, a(x, \bar{x})]$. We say that X* is a locally star shaped if X* is locally star shaped at any $\bar{x} \in X^*$.

1.4 Definition: Let $f : X^* \to R$ be a function, where $X^* \subseteq R^n$ is a locally star shaped set at $\bar{x} \in X^*$, with the corresponding maximum positive number $a(x, \bar{x})$ satisfying the required conditions. We say that f is:

(i_1) semilocally b – pre invex (s/b – pre invex) at $\bar{x}$ if for any $x \in X^*$, there exist a positive number $d(x, \bar{x}) \leq a(x, \bar{x})$ and a function $b : X^* x\, X^* x\, [0\ \ 1] \to R_+$ such that $f(\bar{x} + \lambda(x - \bar{x})) \leq b(x, \bar{x}, \lambda) f(x) + (1 - \lambda\, b(x, \bar{x}, \lambda)) f(\bar{x})$ for $0 < \lambda < d(x, \bar{x}) \lambda\, b(x, \bar{x}, \lambda) \leq 1$).

If f is semilocally b – pre invex at each $\bar{x} \in X^*$ for the same b, then f is said to be semilocally b - preinvex on X*.

(i_2) semilocally quasi b – pre invex (s/q b – pre invex) at $\bar{x}$ if for any $x \in X^0$, there exists a positive number $d(x,\bar{x}) \le a(x,\bar{x})$ and a function $b: X^* x X^* x [0 \ 1] \rightarrow R^+$ such that

$$f(x) \le f(\bar{x})$$
$$0 < \lambda < d(x,\bar{x}) \lambda b(x,\bar{x},\lambda) f[\bar{x} + \lambda(x-\bar{x})] \le b(x,\bar{x},\lambda) f(\bar{x})$$
$$\lambda b(x,\bar{x},\lambda) \le 1$$

If f is semilocally quasi b – pre invex at each $\bar{x} \in X^*$ for the same b, then f is said to be semilocally quasi b - preinvex on X*.

1.5 Definition: [2], [3] Let $f: X^* \rightarrow R$ be a function, where $X^* \subseteq R^n$ is a locally star shaped set at $\bar{x} \in X^*$. We say that f is semi differentiable at $\bar{x}$ if $(df)^+(\bar{x},(x-\bar{x}))$ exists for each $x \in X^*$, where $(df)^+(\bar{x},(x-\bar{x})) = [f(\bar{x}+\lambda(x-\bar{x})) - f(\bar{x})]$ (the right derivative at $\bar{x}$ along the direction $(x-\bar{x})$. If f is semi differentiable at any $\bar{x} \in X^*$, then f is said to be semi differentiable on X*.
Some properties possessed by the semi differentiable functions are given by Kaul and Lyall [50].

1.6 Definition: Let $f: X^* \rightarrow R$ be a semi differentiable function on $X^* \subseteq R^n$. We say that f is semilocally Pseudo b - preinvex (s/p b - preinvex) at $\bar{x} \in X^*$ if

$$(df)^+(\bar{x},(x-\bar{x})) \ge 0 \Rightarrow b(x,\bar{x},\lambda) f(x) \ge b(x,\bar{x},\lambda) f(\bar{x})$$

If f is semilocally pseudo b - preinvex at each $\bar{x} \in X^*$ for the same b, then f is said to be semilocally pseudo b - preinvex on X*.

1.7 Definition: Let $f: X^* \rightarrow R$ be a semi differentiable function o $X^* \subseteq R^n$. We say that f is semilocally Pseudo b - preinvex (s/p b - preinvex) at $\bar{x} \in X^*$ if for each $x \in X^*, x \ne \bar{x}$ we have $\bar{b}(x,\bar{x})[f(x) - f(\bar{x})] > (df)^+(\bar{x},(x-\bar{x}))$

$$\bar{b}(x,\bar{x}) = b(x,\bar{x},\lambda) \qquad (1)$$

1.8 Definition: Let $f: X^* \rightarrow R$ be a semi differentiable function on $X^* \subseteq R^n$. We say that *f* is semi locally strongly pseudo b – pre invex (s/sp b – pre invex) at $\bar{x} \in X^*$ if

$$\bar{b}(x,\bar{x})(df)^+(\bar{x},(x-\bar{x})) \ge 0 \Rightarrow f(x) \ge f(\bar{x})$$

where $\bar{b}(x,\bar{x})$ is defined by (6.1)

If *f* is s/sp b – pre invex at each $\bar{x} \in X^*$ for the same b, then f is said to be s/sp b - preinvex on X*. For $b(x,\bar{x},\lambda) = 1$ these definitions reduce to those of semilocally preinvex, semilocally quasi - preinvex semilocally pseudo - preinvex.

1.9 Definition: (a) $\bar{x}$ is said to be a local minimum solution to problem (NP) if $\bar{x} \in X$ and there exists $\in > 0$ such that $x \in N \in (\bar{x}) \cap X \Rightarrow f(x) \ge f(\bar{x})$

(b) $\bar{x}$ is said to be the minimum solution to problem (NP) if $\bar{x} \in X$ and

$$f(\bar{x}) = \min_{x \in X} f(x)$$

1.10 Definition: We say that g satisfies the generalized Slater's constraint qualification (GSQ) at $\bar{x} \in X$, if g1 is s/pb – pre invex at $\bar{x}$ and there exists $\hat{x} \in X$ such that $g_1(\hat{x}) < 0$.

Sufficient Optimality Theorems

2.1 Theorem: Let $f: X^* \rightarrow R$ be an semi differentiable function on an locally star shaped set X*.

(a) The function f is s/b – pre invex at $\bar{x} \in X^*$ if and only if $(df)^+(\bar{x},\eta(x,\bar{x}))$ exists and $\bar{b}(x,\bar{x})[f(x) - f(\bar{x})] \ge (df)^+(\bar{x},(x-\bar{x}))$.

(b)If *f* is s/q b – pre invex, then $f(x) \le f(\bar{x}) \Rightarrow \bar{b}(x,\bar{x})\ (df)^+(\bar{x},\bar{b}(x,\bar{x})) \le 0$

$$\bar{b}(x,\bar{x}) = \lim_{\lambda \rightarrow 0^+} \bar{b}(x,\bar{x},\lambda) = b(x,\bar{x},\lambda) \text{ and } \lambda b(x,\bar{x},\lambda) \le 1$$

2.2 Sufficient Optimality Criteria:
Consider the nonlinear programming problem (NP)
Minimize f(x)
Subject to: g(x) ≤0, x € X*
where $X^* \subseteq R^n$ is a nonempty η - locally star shaped set and $f : X^* \to R,\ g : X^* \to R^m$ are semi differentiable functions
Let $X = \{x \in X^* / g(x) \le 0\}$ be the set of all feasible solutions to (NP).
Let $N \in (\bar{x})\ \{x \in R^n / \| x - \bar{x} \| < \in\}$

2.3: Theorem: Let $\bar{x} \in X^0$ and let *f* be s/b1 – pre invex at $\bar{x}$ and g be s/b2 – pre invex at $\bar{x}$. If there exists $\bar{u} \in R^m$ such that $(\bar{x}, \bar{u})$ satisfies the conditions

$$(df)^+\ (\bar{x}, (x - \bar{x})) + \bar{u}^T (dg)^+\ (\bar{x}, (x - \bar{x})) \ge 0\ \forall\ x \in X, \qquad (2)$$

$$\bar{u}^T\ g(\bar{x}) = 0 \qquad (3)$$

$$g(\bar{x}) \le 0 \qquad (4)$$

$$\bar{u} \ge 0 \qquad (5)$$

with $\bar{b}_1 (x, \bar{x}) = b_1 (x, \bar{x}, \lambda) > 0$, then $\bar{x}$ is an optimal solution to problem (NP).

2.4 Corollary: Let $\bar{x} \in X^0$ and let f be s/b1 – pre invex at $\bar{x}$ and g be s/b2 – pre invex at $\bar{x}$. If there exists $\bar{u}^0 \in R\ \ and\ \ \bar{u} \in R^m$ such that $\bar{x}, \bar{u}^0, \bar{u}$ satisfy (3) and (4) of theorem 2.3 and the conditions

$$\bar{u}^0 (df)^+ (\bar{x}, (x - \bar{x})) + \bar{u}^T (dg)^+ (\bar{x}, (x - \bar{x})) \ge 0 \,\forall\, x \in X$$

$$(\bar{u}^0, \bar{u}) \ge 0,\ \ (\bar{u}^0, \bar{u}) \ne 0$$

$$\bar{u}^0 > 0$$

with $\bar{b}(x, \bar{x}) = \lim_{\lambda \to 0^+} \bar{b}(x, \bar{x}, \lambda)$, then x is an optimal solution to problem (NP).

2.5 Theorem: Let $\bar{x} \in X^0$, f be s/spb - preinvex and g_I be η - semi differentiable and s/qb – pre invex at $\bar{x}$. If there exists $\bar{u} \in R^m$ such that ($\bar{x}$, $\bar{u}$) satisfy conditions (2) - (5) of theorem 2.3, then $\bar{x}$ is an optimal solution to problem (NP).

2.6. Theorem: Let $\bar{x} \in X^0$, we assume that there exists $\bar{u} \in R^m$ such that at $\bar{x}$, f is s/spb - preinvex, the numerical function $\bar{u}_1\ g_1$ is η - semi differentiable and s/qb - preinvex and such that ($\bar{x}$, $\bar{u}$) satisfies conditions (2) - (5) of Theorem 2.3 Then $\bar{x}$ is an optimal solution to problem (NP).

2.7. Theorem: Let $\bar{x} \in X^0$, we assume that there exists $\bar{u} \in R^m$ such that ($\bar{x}$, $\bar{u}$) satisfies conditions (2) - (5) of Theorem 2.3 and the numerical function $f + \bar{u}_t$ is s/sp b – pre invex at $\bar{x}$. Then $\bar{x}$ is an optimal solution to problem (NP).

Necessary Optimality Criteria

3.1 Lemma: Let $\bar{x} \in X$ be a local minimum solution to (NP). We assume that g_i is continuous at $\bar{x}$ for any $i \in J$, and that f, g1 are semi differentiable at $\bar{x}$. Then the system

$$(df)^+ (\bar{x}, (x - \bar{x})) < 0$$

$$(dg_1)^+ (\bar{x}, (x - \bar{x})) < 0 \text{ has no solution } x \in X^0 .$$

3.2. Theorem: (Fritz John type necessary optimality criteria)

Let us suppose that g_i is continuous at $\bar{x}$ for any $i \in J$. Assume also that $(df)^+ (\bar{x}, (x - \bar{x}))$ and $(dg_1)^+ (\bar{x}, (x - \bar{x}))$ are pre invex functions of x on X*. Which is locally star shaped set at $\bar{x}$. If $\bar{x}$ is a local minimum solution to problem (NP), then there exist $\bar{u}^0 \in R\ \ and\ \ \bar{u} \in R^m$ such that

$$\bar{u}^0 (df)^+ (\bar{x}, (x - \bar{x})) + \bar{u}^T (dg)^+ (\bar{x}, (x - \bar{x})) \ge 0 \,\forall\, x \in X^0$$

$$\bar{u}^T\ g(\bar{x}) = 0,$$

$$(\bar{u}^0, \bar{u}) \ge 0, (\bar{u}^0, \bar{u}) \ne 0$$

3.3. Theorem: (Kuhn - Tucker type necessary optimality criteria)
Let $\bar{x} \in X$ be a local minimum solution to problem (NP) and let g_i be continuous at $\bar{x}$ for $i \in J$ Assume also that $(df)^+(\bar{x},(x-\bar{x}))$ and $(dg_1)^+(\bar{x},(x-\bar{x}))$ be pre invex functions of x on X* an locally star shaped set at $\bar{x}$. If g satisfies GSQ at $\bar{x}$, then there exists $\bar{u} \in R^m$ such that

$$(df)^+(\bar{x},(x-\bar{x}))+\bar{u}^T(dg)^+(\bar{x},(x-\bar{x}))>0 \text{ for all } x \in X^*,$$
$$\bar{u}^T g(\bar{x})=0,$$
$$g(\bar{x}) \le 0, \bar{u} \ge 0.$$

Duality
Relative to the problem (NP) we consider the dual (D)

$$\text{Maximize} \quad \psi(u,y)=f(u)+y^T g(u)$$
$$\text{Subject to} \quad (df)^+(u,(x-u))+y^T(dg)^+(u,(x-u)) \ge 0 \quad \text{for all } x \in X$$
$$y \ge 0, u \in X^0, y \in R^m$$

Where Xo is a nonempty locally star shaped set at any $x \in X^*$ let w denote the set of all feasible solutions to problem.

4.1: Theorem (Weak Duality): Let $\bar{x} \in X$ and $(\bar{u},\bar{y}) \in w$. If f and g are s/b - preinvex on X* with $\bar{b}(\bar{x},\bar{u})=\lim_{\lambda \to 0^+} b(x,\bar{u},\lambda)>0$ then $f(\bar{x}) \ge \psi(\bar{u},\bar{y})$

4.1.1 Corollary: Let $\bar{x} \in X$ and $(\bar{u},\bar{y}) \in w$ such that $f(\bar{x}) \ge \psi(\bar{u},\bar{y})$.
If the hypotheses of Theorem 3.2 are satisfied then $\bar{x}$ and $(\bar{u},\bar{y})$ are the optimal solutions to (P) and (D) respectively

4.2 Theorem (Direct Duality): Let $\bar{x} \in X$ be an optimal solution to (NP), *f* and *g* be semi differentiable at $\bar{x}$ and

(i_1) $(df)^+(\bar{x},(x-\bar{x}))$ and $y^T(dg)^+(\bar{x},(x-\bar{x}))$ are pre invex functions of x on X* an locally star shaped set at $\bar{x}$;

(i_2) g_i ($i \in J$) are continuous at $\bar{x}$;

(i_3) g satisfies the generalized Slater's constraint qualification at $\bar{x}$. Then there exists $\bar{y} \in R^m$ such that $(\bar{x},\bar{y}) \in w$ and $f(\bar{x})=\psi(\bar{x},\bar{y})$. Moreover, if the functions *f* and *g* are s/b - preinvex on Xo and $\bar{b}(x,u)>0$ for all $(u,y) \in w$ then $\bar{x}$ is an optimal solution to (NP) and ($\bar{x}$, $\bar{y}$) is an optimal solution to (WD).

4.3 Theorem (Strict Converse Duality): Let $\bar{x} \in X$ be an optimal solution to (NP), f and g be semi differentiable at x_ and

(i_1) $(df)^+(\bar{x},(x-\bar{x}))$ and $y^T(dg)^+(\bar{x},(x-\bar{x}))$ are pre invex functions of x on X°, an locally star shaped set at $\bar{x}$;

(i_2) g_i ($i \in J$) are continuous at $\bar{x}$;

(i_3) g satisfies the generalized Slater's constraint qualification at $\bar{x}$.

(i_4) g is s/b - preinvex on X°.

If (x*, y*) is an optimal solution of (WD), *f* is semilocally b - preinvex on X° and $\bar{b}(\bar{x},x^*)>0$ then $x^*=\bar{x}$, i.e., x* is an optimal solution to (NP) and $f(\bar{x})=\psi(x^*,y^*)$

Conclusions
We proved semi differentiable B- preinvex function through necessary and sufficient Optimality theorems and also proved by using week and strong duality theorems.

References

Do Van Luu and Le Minh Tung (2002), "Non smooth B- preinvex functions", Vol.27. pp 33-40.
Kaul, R.N. and Lyall,V., (1988) " Non –convex and semi differentiable functions", Indian J.Pure Appl.Math.1912,pp 1167-1179.
P.Kanniappan and S.Pandian, " On Generalized B-vexity in non- linear programming Duality", IJOMAS, Vol.13.,pp.115- 124.
Preda,V.,Stancu-Minasian, I.M. and Batatorescu,A. (1996): " Optimality and duality in non linear programming involving semilocally preinvex and related functions," J. Inform Optim.Sci.17,3, pp 585-596.

Preda,V.,Stancu-Minasian, I.M. (1997): " Duality in multiple objective programming involving semilocally preinvex and related functions. Glasnik Matematicki 32 (52) 1, pp 153-165.
Preda, N.C. , Chancal Singh and Bector,C.R.(1992), " Further properties of B-Vex function", Journal of Infoprmation and Optimization Sciences, 13 (2),195-206.
R.B.Patel (2001)," Duality for non-linear fractional programming involving generalized semi-locally B-Vex function", T.Ind Math. Soc.Vol.68, No.3, 1-4, pp 41-48.
Sudha Gupta, (1995), "Duality in multiobjective non-linear programming involving semilocally B-Vex and related function", Opsearch, Vol.32, No.2.
Suneja S.K. , Singh, C. and Bector,C.R. (1993): " Generalizaiton of preinvex and B-vex functions", J.Optim.Theory Appl. 76 ,3, pp 577-587
Varalakshmi, G. and P.R.S.Reddy (2005): "Duality in multi-objective fractional programming problem involving semilocally b-vexity and related functions, vision 2020, Re strategic role of operational research.
Weir,T. and Mond,B., (1988)" Preinvex functions in multiple objective optimization, J.Math.Anal.Appl.,136,pp 29-38.