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Full Length Research Paper

Optimal Operating Policies of E.P.Q Models for Deteriorating Items with Generalized Pareto Life Time

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Abstract
In this paper we develop and analyse an E.P.Q model with the assumption that the life time of the commodity is random and follows a generalized pareto distribution. It is also assumed that demand is a power function of time. Using the differential equations the instantaneous state of inventory is derived. The total cost function is obtained with suitable cost considerations. Minimizing total cost function, the optimal ordering policies are obtained. The sensitivity analysis of the model with respect to the cost parameters is also obtained. This model is useful in practical situations arising at places like the food, sea food, and vegetable markets, oil industries and photochemical industries.

Keywords: *Generalized pareto, Deteriorating items, lifetime, optimal ordering policies.*

Introduction

The economic order quantity (EOQ) also known as the Wilson lot size) is one of the earliest and most well-known results of Inventory theory (9). In 1915 Harris (5) first derived this formula. Since then, numerous research efforts have been undertaken to extend the basic EOQ model by relaxing various assumptions so as to develop the models that confirms real situations better. The economic production quantity (EPQ) model, which considers the finite replenishment (production) rate, is one of the extending results and is widely is used in industry. Also the EOQ/EPQ models with/without shortages can be found in many literatures and text books (4, 6, 9). To derive the *EOQ*/EPQ formulas with/without shortages that minimize the relevant costs, we note that one convenient way is to use the classical optimization techniques.

Recently much work has been reported in literature regarding perishable inventory models having finite or random life time with various types of demand. They assumed different types of distributions for life time of the commodity. Much work has been reported in literature regarding inventory models with assumption that the lifetime of the commodity is random. When life time of the commodity is random it is to be modelled through ascribing a suitable probability distribution to the model. In order to ascribe an appropriate distribution to the random variable under consideration it is important to observe the process and approximate the imbedded process with the suitable probability model. Taking this concept several researchers assumed different distributions of the life time of the commodity. Ghare and Schrader (1963), Shah and Jaiswal (1977), Cohen (1977), Agarwal (1978), Dave and Shah (1982), Pal (1990), Kalpakam and Sapna (1996), Giri and Chowdary (1999) and others have assumed that the life time of the commodity follows an exponential distribution, Tadikamella (1978) has assumed gamma distribution to the life time of commodity Covert and Philip (1993), Philip (1974), Goel and Agarwal (1980), Hwang and Hwang(1982), Hwang and Choi (1984), Venkata Subbaiah (1999) and others have assumed Weibull distribution to the lifetime of the commodity. K. Nirupamadevi et.al. (2001, 2004) have assumed Weibull distribution to the lifetime of the commodity. However no serious attempt is made to utilize the generalized Pareto distribution for approximating the life time of commodity which is more suitable for items like, natural oil, chemicals which may behave like " peak over threshold" and permits to entrust the information concerning only on the right tail of the distribution of the life time. Among commonly used parametric and Trio-Bi-Parametric functions of the generalized Pareto distribution appears clearly as the best model with generalized Pareto distribution appears clears as the best model, K. Srinivasrao et.al (2005), has developed the model with generalized pareto lifetime and instantaneous replenishment but in many production processes the replenishment is finite. K. Srinivasrao, Kousar Jaha Begum et. al (2007a, 2007b) have developed EPQ for deteriorating items with generalized Pareto life time.

Recently much emphasis is given for inventory model with random lifetime and it also have been reviewed and developed the perishable inventory models with exponential lifetime and their optimal operating strategies inventory model with Gamma distribution for deterioration , inventory models with Weibull distribution for lifetime of the commodity, inventory models with the assumptions that the lifetime of the commodity follows a mixture of Weibull distribution inventory models for deteriorating items, with exponential, Weibull and mixture of Weibull lifetime distributions having seconds sale inventory model for deteriorating items with Weibull rate of decay and finite replenishment.

Earlier researchers have not made any attempt to develop and analyse inventory models with Generalized Pareto distribution, except the work of Srinivasa Rao, et.al (2005) who developed the inventory models with infinite rate of replenishment. But in many production processes the replenishment rate is finite. By considering the above facts an inventory model has been developed and analysed with Generalized Pareto Life time for finite rate of replenishment. The distribution function of the Generalized Pareto distribution (Pickands, Hoskin et al. (1975)) is $F(t, c, a) = 1 - \left(1 - \frac{ct}{a}\right)^{\frac{1}{c}}$

For c = 0 and c = 1, and this distribution reduces to the exponential distribution with mean a, and uniform distribution with range [0, a] respectively. This distribution is extensively used in the analysis of extreme events especially in reliability studies when robustness is required against heavier time or lighter time alternatives to an exponential distribution. Darghi-Noubary (1989) recommends generalized Pareto distribution for use as the distribution of the excess of observed value over an arbitrarily chosen threshold. He pointed out that, "The generalized Pareto distribution arises as a class of limit distribution for the excess over a threshold, as the threshold increases towards the right hand end distribution (tail). The generalized Pareto distribution is more suitable for lifetime distribution for some commodities like, food and vegetables, edible oils, natural gas etc. Using the differential equations the instantaneous state of Inventory is derived. The total cost function is obtained by considering the suitable cost and to minimize the optimal order quantity.

Assumptions and Notations

In this section the assumptions and notations used in this paper are given below.

1. The demand rate is known
2. The lead time is zero
3. Shortages are allowed and fully backlogged
4. A deteriorated item is lost
5. k, finite production rate
6. T, the fixed duration of production cycle is known
7. A, the setup cost for each cycle.

The assumption 7 is made with respective to the producer's perspective and we are minimizing the total production cost.

The life time of commodity is random and follows a Generalized Pareto distribution having probability density function of the form.

$$f(t) = \begin{cases} \frac{1}{a}\left(1 - \frac{ct}{a}\right)^{\frac{1}{c}-1} & c \neq 0 \\ \frac{1}{a} e^{-\frac{t}{a}} & c = 0 \end{cases}$$

The instantaneous rate of deterioration h (t) is

$$h(t) = \frac{1/a\left\{1 - \frac{ct}{a}\right\}^{\frac{1}{c}-1}}{\left(1 - \frac{ct}{a}\right)^{1/c}} = \frac{1}{a - ct}$$

Q: The ordering quantity in one cycle.
C: The cost per unit.
h: Inventory holding cost per unit per unit time.
π: The shortage cost per unit per unit time.

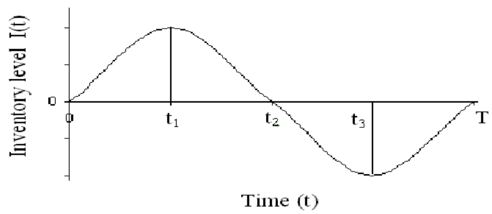
Inventory model with constant demand as power function of time:

In this section demand pattern is considered as uniform throughout the period and demand rate is of form $\lambda(t) = \frac{rt^{\frac{1}{n}-1}}{nT^{1/n}}$,

where n is the index parameter and r is demand size.

Let the ordering quantity in the cycle length T be Q, the cost price of one unit be C, the inventory holding cost per unit time be h, the shortage cost per unit time be □□

The amount of stock is zero at time t = 0. Production starts at t=0 and stops at $t = t_1$. The stock attains a level S at $t = t_1$. During (t_1, t_2) the inventory level gradually decreases mainly to meet up the demand and partly due to deterioration. By this process the stock reaches zero level at $t = t_2$. Now storages occur and accumulate to the level 'P' at $t = t_3$. Production starts again at $t = t_3$ and backlog is cleared at t = T. The cycle then repeats itself after time T. The schematic diagram showing the inventory level over time is given in figure.



Let I (t) be the inventory level of the system at time ($0 \le t \le T$). Then the differential equations describing the instantaneous states of I (t) over the cycle of length T are

$$\frac{d}{dt}I(t)+h(t)I(t)=k-\frac{rt^{\frac{1}{n}-1}}{nT^{1/n}}, \qquad 0\le t\le t_1 \ldots\ldots\ldots\ldots\ldots(3.1)$$

$$\frac{d}{dt}I(t)+h(t)I(t)=-\frac{rt^{\frac{1}{n}-1}}{nT^{1/n}}, \qquad t_1\le t\le t_2 \ldots\ldots\ldots\ldots\ldots\ldots (3.2)$$

$$\frac{d}{dt}I(t)=-\frac{rt^{\frac{1}{n}-1}}{nT^{1/n}}, \qquad t_2\le t\le t_3 \ldots\ldots\ldots\ldots\ldots\ldots (3.3)$$

$$\frac{d}{dt}I(t)=k-\frac{rt^{\frac{1}{n}-1}}{nT^{1/n}}, \qquad t_3\le t\le T \quad \ldots\ldots\ldots\ldots\ldots\ldots (3.4)$$

With the initial conditions I (0) = 0; I (t_2) = 0, I (T) = 0 Substituting h (t) and solving the above differential equations (3.1), (3.2),(3.3) and (3.4), the on hand inventory at time t can be obtained as,

$$I(t)=(a-ct)^{1/c}a^{-1/c}\left\{k\left[t+\frac{t^2}{2a}+\frac{(1+c)t^3}{6a^2}+(1+c)(1+2c)\frac{t^4}{24a^3}\right]\right.$$
$$\left.-\frac{r}{T^{1/n}}\left[t^{1/n}+\frac{t^{1+1/n}}{a}\frac{1}{n+1}+(1+c)\frac{t^{1/n+2}}{2a^2}\frac{1}{1+2n}+(1+c)(1+2c)\frac{t^{1/n+3}}{6a^3}\frac{1}{1+3n}\right]\right\}$$

$$0\le t\le t_1 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3.5)$$

$$I(t)=(a-ct)^{1/c}\frac{ra^{-1/c}}{T^{1/n}}\left[(t_2^{1/n}-t^{1/n})+\frac{(t_2^{1/n+1}-t^{1/n+1})}{(1+n)a}+\frac{(1+c)}{2a^2}\frac{(t_2^{1/n+2}-t^{1/n+2})}{(1+2n)}+\frac{(1+c)(1+2c)}{6a^3}\frac{(t_2^{1/n+3}-t^{1/n+3})}{(1+3n)}\right]$$
$$t_1\le t\le t_2 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (3.6)$$

$$I(t)=-\frac{r}{T^{1/n}}\left[t^{1/n}-t_2^{1/n}\right] \qquad t_2\le t\le t_3 \qquad \ldots\ldots\ldots\ldots\ldots(3.7)$$

$$I(t)=r\left[1-\left(\frac{t}{T}\right)^{1/n}\right]-k(T-t) \qquad t_3\le t\le T \qquad \ldots\ldots\ldots\ldots\ldots\ldots (3.8)$$

The stock loss due to deterioration in the interval (0, T) is

$$L(t) = kt_1 - \frac{rt^{1/n}}{nT^{1/n}}.t_2 \qquad \text{……………………. (3.9)}$$

The back logged demand at time t is

$$B(t) = \frac{r}{T^{1/n}}\left(t^{1/n} - t_2^{1/n}\right). \qquad t_2 \le t \le t_3 \qquad \text{…. ………….(3.10)}$$

The ordering quantity in a cycle of length T is

$$Q = kt_1 + k(T-t_3); \qquad \text{………………………. (3.11)}$$

The total cost per a unit time is sum of the setup cost per a unit time, the unit cost, inventory holding cost, and shortage cost, K (t_1, t_2, t_3, T) is obtained as

$$K(t_1, t_2, t_3, T) = \frac{A}{T} + \frac{CQ}{T} + \frac{h}{T}\left[\int_0^{t_1} I(t)dt + \int_{t_1}^{t_2} I(t)dt\right] + \frac{\pi}{T}\left[\int_{t_2}^{t_3} I(t)dt + \int_{t_3}^{T} I(t)dt\right]$$

Substituting equations (3.5), (3.6), (3.7), (3.8) and (3.11) in the above equation integrating and simplifying by neglecting higher power of 1/a.

$$K(t_1, t_2, t_3, T) = \frac{A}{T} + \frac{Ck(t_1+T-t_3)}{T} + \frac{h}{T}\left\{k\left[\frac{t_1^2}{2} - \frac{t_1^3}{6a} + \frac{t_1^4}{24a^2}(1-2c) - \frac{t_1^5}{120a^3}(2c-1)(3c-1)\right]\right.$$

$$-\frac{r}{T^{1/n}}\left[\frac{nc[2c(1+n)-3n]}{6a^3(1+n)(1+4n)}t_1^{1/n+4} + \frac{nc}{2a^2(1+3n)}t_1^{1/n+3} - \frac{(2c+1)(3c+1)t_2^{1/n+4}}{24a^3(1+4n)} - \frac{t_2^{1/n+3}(1+2c)}{6a^2(1+3n)} - \frac{t_2^{1/n+2}}{2a(1+2n)}\right.$$

$$\left.\left.-\frac{1}{n+1}.t_2^{1/n+1} + \frac{(1+c)(1+2c)t_2^{1/n+3}t_1}{6a^3(1+3n)} - \frac{(1+c)t_2^{1/n+2}t_1^2}{4a^3(1+2n)} - \frac{(1-c)(1-2c)t_2^{1/n}t_1^4}{24a^3} + \frac{(1-c)t_2^{1/n}t_1^3}{6a^2} - \frac{t_2^{1/n}t_1^2}{2a} + t_2^{1/n}t_1\right]\right\}$$

$$+\frac{\pi r}{T^{1/n+1}(n+1)}\left[nt_3^{1/n+1} - (n+1)t_2^{1/n}t_3 + t_2^{1/n+1}\right] + \frac{\pi}{T}\left[\frac{k}{2}(T-t_3)^2 - \frac{r}{n+1}\left(T-(n+1)t_3 + n\frac{t_3^{1/n+1}}{T^{1/n}}\right)\right]$$

……………………………. (3.12)

Differentiating K (t_1, t_2, t_3, T) w.r.t ‘ t_1’ and equating to zero, we get

$$Ck + h\left\{k\left[t_1 - \frac{t_1^2}{2a} + \frac{t_1^3(1-2c)}{6a^2} - \frac{t_1^4(2c-1)(3c-1)}{24a^3}\right] - \frac{r}{T^{1/n}}\left[\left(\frac{c[2c(1+n)-3n]}{6a^3(1+n)}\right)t_1^{1/n+3} + \frac{ct_1^{1/n+2}}{2a^2} + \frac{(1+c)(1+2c)t_2^{1/n+3}}{6a^3(1+3n)}\right.\right.$$

$$-\frac{(1+c)t_2^{1/n+2}t_1}{2a^3(1+2n)} + \frac{(1+c)t_2^{1/n+2}}{2a^2(1+2n)} + \frac{(1-c)t_2^{1/n+1}t_1^2}{2a^3(1+n)} - \frac{t_2^{1/n+1}t_1}{a^2(1+n)} + \frac{t_2^{1/n+1}}{a(1+n)} - \frac{(1-c)(1-2c)t_2^{1/n}t_1^3}{6a^3}$$

$$\left.\left.+\frac{(1-c)}{2a^2}t_2^{1/n}t_1^2 - \frac{t_2^{1/n}}{a}t_1 + t_2^{1/n}\right]\right\} = 0$$

……………………………. (3.13)

Differentiating K (t_1, t_2, t_3, T) w.r.t t2 and equating to zero, we get

$$h\left\{\frac{(1+2c)t_2^{1/n+2}}{6a^2}\left(1+\frac{(3c+1)t_2}{4a}\right) + t_2^{1/n}\left(1+\frac{t_2}{2a}\right) - \frac{(1+c)(1+2c)t_2^{1/n+2}t_1}{6a^3}\right.$$

$$-\frac{(1+c)t_2^{1/n+1}t_1}{2a^2}\left(1-\frac{t_1}{2a}\right) + \frac{t_2^{1/n}t_1^2}{2a^2}\left(1-\frac{(1-c)t_1}{3a}\right)$$

$$\left.-\frac{(1-c)t_2^{1/n-1}t_1^3}{6a^2}\left(1-\frac{(1-2c)t_1}{4a}\right) - t_2^{1/n-1}t_1\left(1-\frac{t_1}{2a}\right) - \frac{t_2^{1/n}.t_1}{a}\right\} + \pi t_2^{1/n-1}(t_2 - t_3) = 0$$

…………………………….(3.14)

By differentiating K (t_1, t_2, t_3, T) w.r.t t_3 and equating to zero, we get

$$Ck + \pi\left[k(T - t_3) - r\frac{(T^{1/n} - t_2^{1/n})}{T^{1/n}}\right] = 0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (3.15)$$

Solving equations (3.13), (3.14), (3.15) for various values of a, c, r, n, C, h, k, π, the optimal values of t_1^*, t_2^*, t_3^* and Q*, K are computed.

Numerical Illustration

Consider the case of deriving the economic production quantity, production down time and production up time for a food processing industry which process sea foods. In this industry the product is of deteriorating nature and the life time is random. After discussion with production managers and workers we collected the data on the life time of commodity and found that it follows a Generalized Pareto Distribution through a frequency curve. Assuming the life time of commodity follows a Generalized Pareto Distribution and using the data collected over a cycle period of 400 hrs (the working time of the industry in three weeks after deducting the time for startup, maintenance, breakdown etc,.), the deteriorating distribution parameters are estimated through the method of maximum likely hood estimation and the estimates are c = 0.2, a = 105. A chi-square test for goodness of fit also carried and found that the Generalized Pareto Distribution gives a good fit to the data. The discussion with production manager and manufactures revealed that the estimates for production cost of a unit , setup cost , holding cost of a unit per unit time and penalty cost of a unit per unit time are C = Rs.3, h = Rs.0.5, π = Rs.0.6, A = Rs. 1200, and T = 400 hrs. The demand parameters are also estimated as r = 550, n = 5, the rate of production per an hour is k = 5 units, with these parameters, using the above model the optimal production schedule is computed and found that the optimal down time is t_1^* = 94.4 hrs, the optimal start-up time t_3^* = 391 hrs, with these two values the economic quantity for cycle is 517 units, and the minimum production cost per unit time is Rs. 14. From these optimal values, the production manager of the unit has continuing production and manufacturing the production scheduling. A sensitivity analysis is also performed for checking the effectiveness of the model with respect to the deteriorating parameters 'a' and c; demand parameters r and n; cost parameters C, h and π; and production rate k on optimum polices for different values of the parameters. The following values for the parameters are considered.

A = 100, 105 ,110 ; c = -1 , 0.2 , 0.25 , 0.3 , 0.35 ; r = 550 ,560 ,570 ; n = 5 ,6 ,8 ,9 ; C = Rs. 3 ,3.5 ,4 ,4.5 ; h = Rs.0.5 ,0.52 ,0.53 ,0.55 ; π = Rs.0.6 , 0.62 , 0.63 , 0.65 ; k = 5 , 6 , 7 ,8 ; T = 400 hrs & A = Rs. 1200.

Using the equations (3.13), (3.14) and (3.15) we obtained the optimal values of t_1, t_2, and t_3. By substituting these values in (3.12) and (3.11) we computed the expected minimum cost per unit time and economic production quantity Q per cycle and presented in table 1.

A careful perusal of table 1 reveals that the deteriorating parameters have a significant influence on optimal production schedule. As the parameter 'a' increases the optimal value of production down time, economic order quantity and minimum cost per unit time are decreasing and production up time is increasing, when other cost and parameters are fixed. With respect to the other deteriorating parameter 'c' the production up time and economic order quantity are increasing for an increase in 'c' and the production down time is decreasing.

With respect to cost parameters there is a significant impact on the optimal values of the production schedule. An increase in holding cost per unit will increase the optimal values of start-up time, shutdown time and economic production quantity. As the shortage cost π increases, the optimal values of the production down time, the production uptime, economic order quantity are increasing, when other parameters remain fixed.

If the demand parameter r increases, the optimal value of t_1 is increasing, where as the optimal values of t_3 is decreasing. With respective the parameter n the optimal values of t_1 and Q are decreasing and optimal value t_3 is increasing for an increase in n. So we conclude that the production manager has to effectively the estimate the parameters of the demand rate for preparing the optimal production schedule.

Inventory model with demand as power function of time and without shortages.

The differential equations governing the instantaneous state of inventory level of the system at time t of the model are

$$\frac{d}{dt}I(t) + h(t)I(t) = k - \frac{rt^{\frac{1}{n}-1}}{nT^{1/n}} \quad , \qquad 0 \le t \le t_1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.1)$$

$$\frac{d}{dt}I(t) + h(t)I(t) = -\frac{rt^{\frac{1}{n}-1}}{nT^{1/n}} \quad , \qquad t_1 \le t \le T \ldots\ldots\ldots\ldots\ldots\ldots (4.2)$$

with the initial conditions I(0)=0, I(T)=0.
The on hand inventory at time t is obtained as,

$$I(t) = (a-ct)^{1/c}.a^{-1/c}\left\{k\left[t+\frac{t^2}{2a}+\frac{(1+c)}{6a^2}t^3+(1+c)(1+2c)\frac{t^4}{24a^3}\right]\right.$$

$$\left.-\frac{r}{T^{1/n}}\left[t^{1/n}+\frac{t^{1+1/n}}{a}\frac{1}{(n+1)}+(1+c)\frac{t^{1/n+2}}{2a^2}\frac{1}{1+2n}+(1+c)(1+2c)\frac{t^{1/n+3}}{6a^3}\frac{1}{1+3n}\right]\right\}$$

$0 \le t \le t_1$ …………………………(4.3)

$$I(t) = (a-ct)^{1/c}\frac{ra^{-1/c}}{T^{1/n}}\left[(T^{1/n}-t^{1/n})+\frac{(T^{1/n+1}-t^{1/n+1})}{(1+n)a}+\frac{(1+c)}{2a^2}\frac{(T^{1/n+2}-t^{1/n+2})}{(1+2n)}\right.$$

$$\left.+\frac{(1+c)(1+2c)}{6a^3}\frac{(T^{1/n+3}-t^{1/n+3})}{(1+3n)}\right]$$

$t_1 \le t \le T$…………………....(4.4)

The stock loss due to deterioration in the interval (0, T) is

$$L(t) = kt_1 - \frac{rt^{\frac{1}{n}}}{nT^{\frac{1}{n}}}T \qquad \text{……………………………(4.5)}$$

The ordering quantity in a cycle of length T is obtained as

$$Q = kt_1 \qquad \text{………………………………….. (4.6)}$$

For obtaining the optimal policies of the perishable inventory model having deterministic demand as power function of time and without shortages, the total cost per unit time k (t_1, T) is obtained as

$$K(t_1,T) = \frac{A}{T} + \frac{CQ}{T}+\frac{h}{T}\int_0^{t_1} I(t)dt+\frac{h}{T}\int_{t_1}^{T} I(t)dt \qquad \text{……………………………….(4.7)}$$

Differentiating K (t_1, T) with respect to t_1 and equating to zero, one can get

$$Ck+h\left\{k\left[t_1-\frac{t_1^2}{2a}+\frac{t_1^3(1-2c)}{6a^2}-\frac{t_1^4(2c-1)(3c-1)}{24a^3}\right]\right.$$

$$-\frac{r}{T^{1/n}}\left[\left(\frac{c[2c(1+n)-3n]}{6a^3(1+n)}\right)t_1^{1/n+3}+\frac{ct_1^{1/n+2}}{2a^2}+\frac{(1+c)(1+2c)T^{1/n+3}}{6a^3(1+3n)}\right.$$

$$-\frac{(1+c)T^{1/n+2}t_1}{2a^3(1+2n)}+\frac{(1+c)T^{1/n+2}}{2a^2(1+2n)}+\frac{(1-c)T^{1/n+1}t_1^2}{2a^3(1+n)}$$

$$\left.\left.-\frac{T^{1/n+1}t_1}{a^2(1+n)}+\frac{T^{1/n+1}}{a(1+n)}-\frac{(1-c)(1-2c)T^{1/n}t_1^3}{6a^3}+\frac{(1-c)T^{1/n}t_1^2}{2a^2}+T^{1/n}\left(1-\frac{t_1}{a}\right)\right]\right\}=0$$

………………………………….(4.8)

For various values of a, c, r and n the optimal values of t^*_1 and Q^* are computed by solving the equations (4.8) and (4.6).

Numerical Illustration of the Model without Shortages

In this section we illustrate the optimal operating policies of the model without shortages by applying it to a food processing industry with the same values for the deteriorating parameters, demand parameters, replenishment parameters, cost value and cycle time given in of section 4. Using the equations (4.8) and (4.6) we computed the optimal shutdown time, optimal production stopping time 181.5 hrs and the economic production quantity is 908 units.

A comparison of the two models under study with the same values of the parameters reveals that the economic production quantity of without shortage model is more than that of with shortages model. And production cost per unit time for the without shortage model is Rs.52, where as for with shortage model it is Rs.14. This shows that without shortages model is more economical than that of with shortages model.

The sensitivity analysis of this model is also carried to study the effect of variation in parameters, costs and cycle time on optimal operating policies. The optimal value of production down time. Economic order quantity and cost per unit time for different values of the parameters and costs are shown in table 2. From table 2 it is observed that the same phenomenon of the earlier model is exhibited for all parameters.

Table 1. Optimal values of t_1*, t_2*, t_3*, K and Q*

a	C	k	H	r	C	N	π	T	t_1*	t_2*	t_3*	K	Q*
100	0.2	5	0.5	550	3	5	0.6	400	94.676	201.335	390.888	17.313	518.9395
105									94.4526	203.228	391.0677	16.932	516.92
110									94.238	205.005	391.235	16.5796	515.015
100	0.25	5	0.5	550	3	5	0.6	400	96.517	200.4367	390.8023	17.324	528.5767
	0.3								98.763	199.883	390.749	17.3033	540.068
	0.35								101.590	199.803	390.741	17.237	554.245
100	0.2	6	0.5	550	3	5	0.6	400	80.414	191.567	392.44	21.624	527.793
		7							69.5369	184.104	393.705	24.988	530.820
		8							61.0537	178.281	394.740	27.675	531.509
100	0.2	5	0.52	550	3	5	0.6	400	94.175	199.170	390.68	17.291	517.47
			0.53						93.9325	198.1205	390.5799	17.227	516.763
			0.55						93.459	196.083	390.3825	17.2403	515.38
100	0.2	5	0.5	550	3	5	0.6	400	94.676	201.335	390.888	17.313	518.9395
				560					96.154	202.345	390. 729	17.12	527.12
				570					97.615	203.344	390.5725	16.919	535.216
100	0.2	5	0.5	550	3.5	5	0.6	400	93.462	200.65	391.656	17.955	509.028
					4				92.261	199.98	392.425	18.585	499.176
					4.5				91.07	199.32	393.195	19.263	489.38
100	0.2	5	0.5	550	3	6	0.6	400	91.122	199.297	392.94	12.763	490.904
						8			86.306	196.504	395.648	6.526	453.289
						9			84.603	195.509	396.589	4.29	440.07
100	0.2	5	0.5	550	3	5	0.62	400	95.3377	203.285	390.911	17.669	522.12
							0.63		95.6634	204.2401	390.925	17.843	523.6915
							0.65		96.3047	206.109	390.95	18.1874	526.153
105	-1	5	0.5	550	3	5	0.6	400	87.44762	209.7924	391.6804	16.831	478.8108
110									85.94079	211.6364	391.84981	16.47413	470.4548
115									85.177	213.6020 8	392.029	16.13127	465.7476
100	-0.9	5	0.5	550	3	5	0.6	400	86.59594	211.142	391.8045	16.944	473.9571
	-0.8								84.87407	214.5348	392.1136	16.6918	463.802
	-0.6								84.140	219.9034	392.594	5	457.726
												16.3384	
100	-1	6	0.5	550	3	5	0.6	400	117.5535	222.3467	394.842	23.3468	736.2674
		8							125.058	226.798	397.624	3	1019.4
		9							126.944	227.97	398.500	36.44	1156
												43.1317	
100	-1	5	0.55	550	3	5	0.6	400	105.366	211.125	391.803	17.294	567.817
			0.56						105.579	210.375	391.734	17.3017	569.228
			0.58						105.9857	208.919	391.599	17.311	571.929
100	-1	6	0.5	560	3	5	0.6	400	116.823	221.9305	394.6265	23.0236	733.183
				580					115.2302	221.0313	394.18	12.388	726.263
				600					113.4024	220.0156	393.731	21.7714	718.024
100	-1	5	0.5	550	4	5	0.6	400	109.193	218.0613	394.097	18.6506	575.480
					5				112.465	220.1411	395.94	20.099	582.578
					6				115.0538	221.88	397.769	21.56	586.419
100	-1	5	0.5	550	3	6	0.6	400	110.1317	218.61	394.463	12.959	578.34
						8			115.587	221.99	397.19	7.349	591.966
						9			117.2278	223.031	398.08	5.405	595.703
100	-1	5	0.5	550	3	5	0.61	400	103.56	215.602	392.128	17.37	559.177
							0.62		102.58	216.0429	392.088	17.529	553.96
							0.63		102.088	216.4238	392.04	17.679	550.213

Table 2. Optimal values of t_1*, K, Q*

A	c	H	r	K	C	N	T	t_1^*	K	Q^*
100	0.2	0.5	550	5	3	5	400	185.028	57.296	925.141
105								181.559	54.932	907.793
110								177.774	52.891	888.868
100	0.25	0.5	550	5	3	5	400	195.046	58.098	975.231
	0.3							207.335	57.729	1037.0
	0.35							223.426	55.453	1117.0
100	0.2	0.55	550	5	3	5	400	185.466	62.031	927.331
		0.6						185.831	63.765	929.156
		0.65						186.14	71.497	930.702
100	0.2	0.5	560	5	3	5	400	186.052	57.124	930.261
			580					187.991	56.756	939.956
			600					189.796	56.359	948.982
100	0.2	0.5	550	6	3	5	400	173.566	69.49	1041
				7				162.346	80.659	1136
				8				151.652	90.836	1213
100	0.2	0.5	550	5	4	5	400	183.425	59.599	917.125
					5			181.826	61.882	909.13
					6			180.231	64.145	901.157
100	0.2	0.5	550	5	3	6	400	173.202	50.47	866.011
						8		155.761	40.004	778.806
						9		149.085	35.91	745.427
100	-0.2	0.5	550	5	3	5	400	137.084	40.069	685.422
105								134.377	39.21	671.886
110								132.119	38.375	660.597
100	-.19	0.5	550	5	3	5	400	137.885	40.444	689.423
	-.18							138.702	40.83	693.511
	-.15							140.389	41.633	701.944
100	-0.2	0.55	550	5	3	5	400	137.675	43.26	688.373
		0.6						138.171	46.45	690.853
		0.65						138.594	49.638	692.968
100	-0.2	0.5	560	5	3	5	400	138.568	39.931	692.839
			580					140.031	39.782	700.155
			600					144.3	39.261	721.502
100	-0.2	0.5	550	6	3	5	400	122.488	48.198	734.931
				8				101.299	61.039	810.393
				9				93.317	66.209	839.854
100	-0.2	0.5	550	5	4	5	400	134.965	41.769	674.824
					5			132.911	43.443	664.553
					6			130.916	45.092	654.578
100	-0.2	0.5	550	5	3	6	400	127.445	33.373	637.226
						7		120.367	28.088	601.835
						8		114.875	23.817	574.374

Table 3: Sensitivity analysis with shortages

Variation in parameters		Percentage change in parameters						
		-15	-10	-5	0	5	10	15
A	K	18.67057	18.17617	17.7259	17.313	16.93229	16.57967	16.25185
	Q	525.02811	523.09095	521.01953	518.93954	516.92472	515.01509	513.22959
C	K	17.29271	17.30045	17.30723	17.313	17.31772	17.32133	17.32378
	Q	513.83527	515.4873	517.18784	518.93954	520.74531	522.60829	524.53194
K	K	13.22189	14.68229	16.04326	17.313	18.49921	19.609	20.64889
	Q	505.2078	510.74121	515.25861	518.93954	521.92879	524.34223	526.27244
H	K	17.28656	17.31543	17.3237	17.313	17.28477	17.2403	17.18071
	Q	525.01028	522.88095	520.85911	518.93954	517.11671	515.38498	513.73883
R	K	18.44719	18.16132	17.78149	17.313	16.76101	16.13057	15.42659
	Q	448.05589	472.32905	495.9658	518.93954	541.22417	562.79559	583.6319
C	K	16.72415	16.92156	17.11784	17.313	17.50704	17.69997	17.89178
	Q	527.91148	524.91538	521.92474	518.93954	515.95978	512.98543	510.01647
N	K	21.75355	20.15136	18.67597	17.313	16.05016	14.87687	13.78397
	Q	546.7895	536.68634	527.43712	518.93954	511.10666	503.86417	497.1483
∏	K	15.59351	16.18941	16.762	17.313	17.84396	18.35623	18.85104
	Q	503.32594	508.784	513.97859	518.93954	523.69154	528.2552	532.64797

Table4: Sensitivity analysis without shortages

Variation in parameters		Percentage change in parameters						
		-15	-10	-5	0	5	10	15
A	K	67.87676	63.5503	60.10843	57.296	54.93247	52.89064	51.083
	Q	960.76341	952.17929	940.15286	925.141	907.79306	888.86752	869.1296
C	K	56.43737	56.74887	57.03587	57.296	57.52687	57.72561	57.88925
	Q	898.85012	907.35197	916.10948	925.141	934.46789	944.1126	954.10127
K	K	47.47832	50.8142	54.08695	57.296	60.44108	63.52182	66.5313
	Q	822.72876	858.35673	892.4896	925.141	956.33019	986.08167	1014.43
H	K	50.19003	52.55929	54.92793	57.296	59.6637	62.03097	64.3979
	Q	920.89511	922.46712	923.87428	925.141	926.2879	927.33066	928.28301
R	K	58.34259	58.07791	57.72459	57.296	56.80327	56.25536	55.65982
	Q	874.54306	893.25077	910.13507	925.141	938.782	951.14099	962.37608
C	K	56.25323	56.60128	56.94889	57.296	57.64275	57.989	58.3348
	Q	928.75451	927.54967	926.34524	925.141	923.93761	922.73443	921.53168
N	K	63.37632	61.24601	59.22127	57.296	55.46407	53.71936	51.05627
	Q	980.25505	960.6141	942.29236	925.141	909.03594	893.87076	879.55524

Conclusion

This paper proposes a continuous production Inventory model for deteriorating items with and without shortages. Here we constrained deterioration of economic production quantity, production down time, and production up time, the case of generalized Pareto rate of decay and time dependent demand. The Generalized Pareto Distribution used for life time of commodity also includes uniform and exponential distributions as particular cases. We have utilized the differential equations and unconstrained optimal techniques under stochastic environment to obtain the solution of the model. The utility of the models in food processing industry is illustrated through a real time data. It is observed that the deteriorating parameters have significant effect on the optimal operating policies of the models. The model with shortages is much economical than without shortages.

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References

Aggarwal, S.P. (1978).A note of an order level inventory model for a system with constant rate of deterioration, Opsearch, Vol 15(4), 184-187.

Aggarwal, S.P. (1979), a note on an order level lot size inventory model for deteriorating items. AIIE Transaction, 11, 344-346.

Aggarwal, S.P. and Goel, V.P. (1980). Pricing and ordering policy with general Weibull rate of deteriorating inventory, Indian Journal of Pure Applied Mathematics, Vol. 11: 5, 618-627.

Aggrawal, S.P. and Goel, V.P. (1982). Order level inventory system with demand pattern for deteriorating items, Econ. Comp. Econ. Cybernet, Stud. Res., Vol. 3, 57-69.

Aggrawal S.P. and Goel, V.P. (1984). Order level inventory system with demand pattern for deteriorating items, Operation Research in Managerial Systems, 176-187.

Chowdhury, M.R. and Chaudhury, K.S. (1983). An order level inventory model for deteriorating items with finite rate of replenishment, Opsearch , Vol. 20, 99-106.

Cohen, M.A. (1976), Analysis of single critical number ordering policies for perishable inventories, Operat. Res., 24, 726-741.

Covert, R. P. and Philip, G.C. (1973), A EOQ model for items with Weibull distribution, AIIE TRAN, 323-326.

]ave, U. and Shah, Y.K. (1982), A probabilistic inventory modal for deteriorating items with lead time equal to one scheduling period, EJOR, 9, 281-282.

Ghare, P.M. and Scharader, G.F (1963), A model for exponentially decaying inventories, J. Indust. Engr., 14,238-243.

Girl, B.C. and Chaudhuri, K.S. (1998), Deterministic models of perishable inventory withstock dependent rate and nonlinear holding cost, EJOR., 105,467-474.

Goel, Vijaya P. (1980). Inventory model with a variable rate of deterioration. Journal of Mathematical Sciences, 14, 5-11.

Goyal, S.K. and Giri, B.C. (2001), Invited review of recent trends in modelling of Deteriorating inventory, EJOR., 134,1-16.

Hang and Hang (1982), An EPQ model for deteriorating items under LIFO Policy, J. Operat. Res. Soc., 25, 48-57.

Mathew, J (2002), some perishable Inventory models with constant rate of replenishment, Ph. D Thesis, Andhra University, Visakhapatnam.

Kalpakam, S and Sapna, KP. (1996), A lost sales (S-I, S) perishable inventory system with renewal demand, Naval. Res. Logistics, 43, 129-142.

Kalpakam, S. and Sapana, K.P. (1996a). An (S,S) Perishable system with arbitrary distributed lead times, Opsearch, Vol. 33-1-19.

Madhavi, S (2002), Some Inventory models for perishable items with seconds sale, Ph.D Thesis, Andhra University, Visakhapatnam.

Mathew, J. (2002), Some perishable Inventory models with constant rate of replenishment, Ph.D Thesis, Andhra University, Visakhapatnam.

Mishra, R.B. (1975). Optimum lot size model for system with deteriorating inventory. International journal of Production of Research, 13: 495-505.

Naddor, E. (1966). Inventory systems. John Wiley, New York.

Nahmias, S. (1982), Perishable inventory theory: A review, Oper. Res., 30, 4,680-708.

Nirupama Devi,. K. (2000), Perishable Inventory models with mixture of Weibull distributions having demand has power junction of time, Ph.D Thesis, Andhra University, Visakhapatnam.

Pal, M. (1990), an inventory model for deteriorating items when demand is random, Cal. Statist, Assco. Bull., 39, 201-207.

Philip, G.C. (1974), a generalized EOQ model for items with Weibull distribution Deterioration, AIIE. Trans., 6, 159-162.

Raafat, F. (1991), Survey of literature on continuously deteriorating inventory models, J.Operat. Res. Soc., 42, 27-37.

Shah, Y. and Jaiswal, M.C (1977), An order level inventory model for a system with Constant rate of deterioration, Opsearch, 14, 174-184.

Tadikamalla, P.R (1978), An EOQ inventory model for items with gamma distributed deterioration. AIlE .Trans., 10,100-103.

Venkatasubaiah, K. et al (1999), Inventory model with stock dependent demand and Weibu11 rate of deterioration. Proceedings of XIX Annual conference of ISPS, India.