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Full Length Research Paper

Generalized Linear Acceleration and Linear Velocity for a Particle of Non zero Mass in a Static Homogeneous Spherical Distribution of Mass

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Abstract
In this paper, we derived the generalized linear acceleration and linear velocity of a particle of nonzero mass in a static homogeneous spherical distribution of mass. The result shows that both the acceleration and linear velocity contain time derivation of the position coordinate and post Newtonian correction to the order of c^2 to the pure Newtonian acceleration and velocity.

Keywords: *Static homogeneous spherical distribution of mass, Post Newtonian correction, Linear acceleration tensor.*

Introduction

The Gravitational force is one of the fundamental forces in nature alongside electromagnetic and nuclear forces. Gravitational force governs the motion of planets, moon and the galaxies is their respective orbits.

The two major theories of gravitation include the Newtonian dynamical theory of gravitation , which explains the manifestation of all interactions in nature through a force. Newton's theory was successful in explaining the gravitation phenomena on the surface of the earth and experimental facts about the solar system [3] and Einstein geometric theory of gravitation published in 1916 called General relativity. In his theory, he generalized special relativity and Newton's law of universal gravitation, providing a unified description of gravity as a geometric property of space and time or space time. Einstein showed through a system of partial differential equation (called Einstein field equations) how the curvature of space time is directly related to the energy and momentum of whatever matter and radiation are present [4]. These equations are given as:

$$R_{ab} - \frac{1}{2} R g_{ab} = \frac{8\pi G}{R_{ab}\, c^4} T_{ab} \qquad (1)$$

The left hand side of the equation is the *Einstein tensor, a* specific divergence-free combination of the Ricci tensor R_{ab} and the metric tensor. On the right hand side, T_{ab} is the energy –momentum tensor and $\frac{8\pi G}{c^4}$ is the proportionality constant with G the gravitational constant and c the speed of light [4]

Theory

The well know tensorial Riemann's Geodesic Equations of motion for particles of nonzero rest masses in gravitational fields is given (2) by

$$\frac{\partial}{\partial\tau}(M_o\, U^{\alpha}) = M_o \frac{\partial}{\partial\tau}(U^{\alpha}) = M_0 a^{\alpha} = 0 \qquad (2)$$

Where; $\frac{\partial}{\partial\tau}$ is covariate differentiation with respect of proper time, and M_o is rest mass, u^{α} is linear velocity tensor and a^{α} is the four dimensional "linear acceleration tensor" given by

$$a^{\alpha} = \ddot{x}^{\alpha} + \Gamma^{\alpha}_{\mu\upsilon}\dot{x}^{\mu}\dot{x}^{\upsilon} \qquad (3)$$

x^{α} is space time coordinate tensor, $\Gamma^{\alpha}_{\mu\nu}$ is the Christoffel symbol of the second kind and a dot denotes one differentiation with respect to proper time.

It follow by definition that the vector a corresponding to the linear acceleration tensor is given by [1,2]

$$a = a_{x^0}, a_u, a_v a_w \quad (4)$$

Where;

$$a_{x^0} = g_{00}^{1/2} a^0 \quad (5)$$

$$a_u = g_{11}^{1/2} a^1 \quad (6)$$

$$a_v = g_{22}^{1/2} a^2 \quad (7)$$

$$a_w = g_{33}^{1/2} a^3 \quad (8)$$

which is henceforth called *linear acceleration vector in gravitation fields.*

It therefore follows that the tensorial Riemann's Geodesic equation of motion given in equation (1) can be written equivalently as a vector equation given by

$$M_o a = 0 \quad (9)$$

Where; M_o is the rest mass of the particle. Equation (9) is henceforth called *the vectorial Riemann's Geodesic Equation* of motion for particles of nonzero rest masses in gravitational field.

It is well known that the Schwarz-child's metric tensor in the gravitational field of a static homogeneous spherical mass distribution in Einstein spherical polar coordinate is given by [2]

$$g_{00} = \left(1 + \frac{2}{c^2} f\right) \quad (10)$$

$$g_{11} = \left(1 + \frac{2}{c^2} f\right)^{-1} \quad (11)$$

$$g_{22} = r^2 \quad (12)$$

$$g_{33} = r^2 \sin^2 \theta \quad (13)$$

$$g_{\mu\nu} = 0 \quad ; \text{otherwise} \quad (14)$$

and

$$x^0 = ct \quad (15)$$

substituting (10) – (13) into (5) – (8), correspondingly, we have

$$a_{x^0} = \left(1 + \frac{2}{c^2} f\right)^{\frac{1}{2}} a^0 \quad (16)$$

$$a_u = \left(1 + \frac{2}{c^2} f\right)^{\frac{-1}{2}} a^1 \quad (17)$$

$$a_v = (r^2)^{\frac{1}{2}} a^2 \quad (18)$$

$$a_w = (r^2 sin^2 \theta)^{\frac{1}{2}} a^3 \quad (19)$$

where the superscripts 0,1,2,3 are time derivative of position coordinate .

Equations (16) – (19) are the linear acceleration tensor of a particle of nonzero rest mass in a gravitational field of a static homogeneous spherical mass distribution in Einstein spherical polar coordinates.

Similarly, it may be noted that by definition in the theory of Tensor Analysis, the physically measurable and meaningful vector corresponding to the velocity tensor, denoted by U is given [2]

$$\underline{U} = (U_u, U_v, U_w, U_{x^0}) \quad (20)$$

where,

$$U_u = g_{11}^{\frac{1}{2}} U^1 \quad (21)$$

$$U_v = g_{22}^{\frac{1}{2}} U^2 \quad (22)$$

$$U_w = g_{33}^{\frac{1}{2}} U^3 \quad (23)$$

$$U_{x^0} = g_{00}^{\frac{1}{2}} U^0 \quad (24)$$

Where; the superscripts; 0,1,2,3, denote time derivative of position coordinate. It is henceforth called the *linear velocity four-dimensional vector in a gravitational field.*

Equation (21) - (24) may be explicitly written for a particle of nonzero test mass in a gravitational field of a static homogeneous spherical mass distribution in Einstein spherical polar coordinates as,

$$U_u = \left(1 + \frac{2}{c^2} f\right)^{\frac{-1}{2}} U^1 \quad (25)$$

$$U_v = (r^2)^{\frac{1}{2}} U^2 \quad (26)$$

$$U_w = (r^2 sin^2 \theta)^{\frac{1}{2}} U^3 \quad (27)$$

$$U_{x^0} = \left(1 + \frac{2}{c^2} f\right)^{\frac{1}{2}} U^0 \quad (28)$$

Discussion and Conclusion

The immediate consequence of the result in this research is that for a particle of nonzero rest mass is a gravitational field of a static homogeneous spherical mass distribution in Einstein spherical polar coordinates, both the linear acceleration and linear velocity contain at least one time derivative of space position coordinate and also contains post Newtonian corrections of all orders of c^{-2} to the pure Newton's linear acceleration and velocity.

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