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Discovering Pricing of European Options under Skewed Brownian motion

Tuhin Mukherjee[1*] and Tushar Kanti Dey[2]

[1*]*Assistant Professor, Department of Business Administration, University of Kalyani, West Bengal, India.*
[2]*University of Kalyani, West Bengal, India.*

**Corresponding author: Tuhin Mukherjee*

Abstract
Usually stock price moves along Brownian motion. Depending on the nature of this price movement, stock prices and its derivatives can be modelled. In this paper, such an attempt has been made where Brownian motion has been skewed to develop a newoption pricing model for European options (both call and put types, which are commonly used in Indian market). Concept of risk neutral probability has been used for this purpose. Finally, the newly proposed model is tested empirically in Indian context to establish its superiority against traditional Black ScholesOption Pricing model in continuous time finance.

Keywords: *Option, Geometric Brownian Motion, Risk neutral probability, Continuous time finance.*

Introduction to the Concept of Risk Neutral Valuation in Probability Space

Stock price S_t is definitely a function of time t but not deterministic function. It is stochastic function of t. Hence it changes with t in an unknown way (Merton, 1973). In other words, S_t is a random variable and $\{ S_t \}_{t>=0}$ is a stochastic process. Now option price is a function of stock price. Hence $\{ C_t \}_{t>=0}$ is also a stochastic process. Hence their differentiations, integrations all are not from simple calculus but from stochastic calculus of advanced mathematics. Thus tools from such advanced level of mathematics can be easily applied to further analyse the behaviour of option prices. In this section, we are going to discuss such a concept known as risk neutral valuation, which is extensively used in subsequent sections of our paper. It is clear from the literature that Stochastic Differential Equation (SDE) of famous BS model is free from μ (the expected rate of return of the stock). This is the only parameter which could relate risk preference of investors. Due to its elimination, BS model actually states that during option pricing, without loss of generality, we can easily assume that world is risk neutral (where all individuals are indifferent to risk). In turn, it says that in the context of option pricing problem, we can assume that risk aversion of all investors is same. The resulting option price will be correct not only in such hypothetical risk neutral world, but also in the market of real world (Rubinstein, 1985).

BS model assumes that market is complete. So there exist a unique Equivalent Martingale Measure (EMM) P^* under which discounted stock price is martingale. Girsanov's Theorem (1960) helps us to find such P^*. Actually this theorem describes how the dynamics of stochastic processes change when the original measure is changed to an equivalent probability measure. A study by Holden (1996) nicely analysed its usefulness. Now, following Cox and Ross (1976) the time t price of a European call option (which is nothing but a typical contingent claim) is given by $C_t= e^{-r (T-t)} E^* [(S_T - K)^+ | F_t]$. As $\{S_t\}$ is adapted to filtration $\{F_t\}$, so we get $C_t = e^{-r (T-t)} E^* [(S_T - K)^+ | S_t] = e^{-r (T-t)} E^* [Max \{(S_te^x - K), 0\} | F_t]$, where x has a normal distribution with mean $(r - \sigma^2/2)(T-t)$ and variance $\sigma^2(T-t)$. If we denote p(x) as the density function of x then we have $C_t = e^{-r (T-t)} I (-\infty,\infty ; (S_te^x - K)) p(x) dx$
$= e^{-r (T-t)} I (\ln (K/S_t), \infty; (S_te^x - K)) p(x) dx$
$= S_t N (d_1) – K e^{-r (T-t)} N (d_2)$, which is the explicit form of BS model as obtained in the previous section of this paper. Thus under risk neutral valuation, we can also get BS formula directly, without solving any SDE. In this paper, we are going to use this risk neutral valuation concept (where both stock price and bond price grow in the same risk free rate) to derive the generalized BS model under some newly introduced assumption of skew Brownian motion.

Brief Review of Relevant Literature

In order to resolve the mispricing problems of the BS model (1973) due to distributional assumptions.alternate underlying price processes have been proposed since long time past. Non-Gaussian processes like hyperbolic Levy motion was used by Eberlain and Keller (1995). In 1997, Corrado and Su provides evidence that BS model produces mispricing of options due to the non-skew assumptions of underlying stock prices.From subsequent studies like Bingham and Keisel (2001), Elliott and Hoek (2001), we find our motivation to use Skewed Brownian motion in the underlying stock price process. This Skewed Brownian motion was introduced by Ito and McKean (1965) but its properties were analyzed in detail by Portenko (1976). According to,Rogers and Satchell(2000), such Skewed Brownian motion is a diffusion process which is characterized by a parameter α in [0,1]. In terms of a standard Brownian motion $\{W_t\}$, we can define a Skewed Brownian motion $\{X_t\}$ as follows:
$X_t = |W_t|$ with probability α
$= - |W_t|$ with probability 1-α

Thus an excursion from zero is more likely to be positive when $\alpha > 0.5$ and negative when $\alpha < 0.5$. At $\alpha = 0.5$, it reduces to standard Brownian motion.

Assumptions of our new option pricing model under Skewed Brownian motion

We are assuming that stock prices are following Skewed Brownian motion instead of geometric Brownian motion. Hence, in contrast to BS model, in our case stock prices does not follow lognormal distribution and stock return is not the normal too. However, other important assumptions of BS model are kept as it were. Thus Volatility of the stock is taken constant over [0, T]. We are mainly considering stocks which are paying no dividends during the life [0, T] of the options. Transaction cost is assumed to be absent. Our market is perfectly liquid and frictionless. Hence any number of stocks and options can be bought or sold. Limitless borrowing is also allowed. In addition, we are also allowing short selling in stocks and options. Primarily this paper considers European type of Options.

Formulation of our new option pricing model under Skewed Brownian motion

According to Corns and Satchell (2007), the stochastic process $\{X_t\}_{t>=0}$ defined b $X_t = (1-\delta^2)^{1/2} W_{1,t} + \delta |W_{2,t}|$ is distributed as an Ito-McKean (1965) Skewed Brownian motion. In this paper, we are taking such alternate form of Skewed Brownian motion for option pricing. This section attempts to derive a closed form solution for a plain vanilla European call option as follows.Due to space limitation and to be compact, we are only reporting the end results only, without detailing the long and complicated mathematical derivation.

In view of the above, we are considering that underlying stock prices follow a stochastic process $\{S_t\}$ given by $S_t = S_0 \exp(\mu t + \sigma X_t)$. The results found in Arellano Valle and Azzalini (2006) helps us to find the density of X_t as

$$P(X_t € dz) = (2/t)\, \phi\, (X_t/(t)^{1/2})\, N[(\delta/(1-\delta^2)^{1/2})(X_t/(t)^{1/2})]\, dz.$$

Therefore European call option price in any time t in [0,T) under risk neutral probability measure P^* is given by $C_t = e^{-r(T-t)} E^* [(S_T - K)^+ | F_t]$. Following the approach of Corns and Satchell (2007), it is easy to evaluate this expectation in continuous time finance and we did found it on simple calculation as $C_t = [2N(\delta\sigma(T-t)^{1/2})]^{-1} S_t\Psi_1(b) - Ke^{-r(T-t)} \Psi_2(b)$. This is the newly formed expression of generalized BS model by means of a single parameter δ in (-1, 1). In particular when $\delta = 0$, then it reduces to the traditional BS model formula.

Empirical Analysis in Indian Context

In this section, we are going compare the performance of our newly proposed option pricing model under SkewedBrownian motion with that of traditional BS model. To do so, we are considering option price data from NSE, India. This paper concentrates on a particular equity stock ACC for the month of June, 2013. With respect to expiry date 27th June, 2013, European call option prices (only closing price from 5th June, 2013 to 26th June, 2013) for ACC are collected against six different exercise prices (with step value of □20) like □1180, □1200, □1220, □1240, □1260 and □1280. Hence we got 16 days closing prices X 6 exercise prices = 96 data points for our empirical analysis.

Now, using our newly formulated model as well as traditional BS model, we priced corresponding European call options and observed their error terms with respect to market data. The entire calculation is done using SPSS but here we are only reporting the findings summary, without including the several tables of numerical figures.

To test the normality, we used non-parametric Chi-square text of goodness of fit (an approximate test using method of likelihood ratio testing), and it was found that error terms of both the samples can be reasonably accepted as from normal populations.
In the next stage, this paper attempted to compare means and variances of two normal populations (using the method of likelihood ratio testing). In the first phase we considered the null hypothesis as H_o: (Population standard deviations are equal) against no alternative. Thus it is tested by means of a two tailed F-test. Null hypothesis is accepted at 5% level of significance. So it became reasonable to assume that population variances are equal but that is unknown to us. Hence this paper came to the phase of comparing population means. As mean error m_1 from sample of traditional BS model was found greater than that m_2 of our modified BS model, so in this case, we considered the null hypothesis H_o: $(m_1 = m_2)$ against the alternate hypothesis H_A : $(m_1 > m_2)$. For this right-tailed t-test, the value of t-statistic falls within the critical region of 5% level but outside that of 1% level. In other word, we found that the value of t-statistic is significant and we have some reasons for rejecting the null hypothesis. So we believe statistically that mean error of traditional BS model is generally more than that of our newly proposed model under Skewed Brownian motion. Such findings establish the merit of our model construction.

Conclusion

In this paper, option pricing problem is addressed. We have attempted to revise pricing mechanism under an alternate stochastic process. The primary assumption of geometric Brownian motion in BS model is replaced by Skewed Brownian motion. The newly proposed model became a generalized model, whose particular case is BS model. Finally, our newly proposed generalized model is empirically tested against traditional approach and found to be effective in pricing of options (both put and call of European style) in Indian context. Further research investigation can be done by incorporating other features like jump discontinuity, variable volatility etc in our newly proposed option pricing model.

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Appendix: (Notations used in this paper)

S_t = Stock price in time t. C_t = European call option price in time t. W_t = Standard Brownian motion. $W_{1,t}$ and $W_{2,t}$ = Two independent standard Brownian motions. F_t = Filtration .σ = Stock volatility. r = Risk free interest rate compounded continuously. T= Expiry time of European call option. N(x)= Cumulative distribution function of Standard normal distribution. ϕ(x) = Probability density function of Standard normal distribution. $(S_T - K)^+ = \text{Max}\ \{(S_T - K), 0\}$ $d_1(S_t, (T-t)) = [\ln(S_t/K) + (r+\sigma^2/2)(T-t)] / \sigma(T-t)$. $d_2(S_t, (T-t)) = d_1 - \sigma(T-t)$. $b = [\ln(K/S_t) - \{(r-\sigma^2/2)(T-t) - \ln(2\ N(\delta\sigma(T-t)^{1/2}))\}] / \sigma(T-t)^{1/2}$. δ = A real number in (-1,1). I(p,q; f(x))dx = Definite integral of f(x) from x=p to x=q i.e. $\int_p^q f(x)\,dx$. $\lambda = \delta/(1-\delta^2)^{1/2}$. $\Psi_1(b) = 2\ I(b, \infty; f(s))ds$, where $f(s) = I(-\infty, \lambda s; \phi(s-\sigma(T-t)^{1/2})\phi(u))du$. $\Psi_2(b) = 2\ I(-\infty,-b;f(s))ds$, where $f(s) = I(-\infty,-\lambda s; \phi(s)\phi(u))du$.