

Full Length Research Paper**Efficiency Estimates of Indian Commercial Banks – An Empirical Approach**S.Chandra Babu ^{1*}, Vijaya Kumar. Kandunuru ², N.Ramesh Kumar ³, and P.Balasidamuni⁴¹ Lecturer in Statistics, NPS Government Degree College for Women, Chittoor.² Lecturer in Statistics, SGS Arts College, T T D, Tirupati.³ Lecturer in Statistics, SV Arts College, T T D, Tirupati.⁴ Retired Professor of Statistics, S.V. University, Tirupati.*** Corresponding author: S.Chandra Babu****Abstract**

In this paper, we propose a method to estimate the efficiency of commercial banks empirically. Banks are considered as Decision Making Units (DMU) responsible for converting inputs into outputs. The inputs identified by this study are: 1.Input Extended 2.Operating expenses 3.Deposits and 4.Investments and the outputs are: 1.Interest Income 2.Other Income. Efficiency is estimated in terms of input requirements to produce certain output and possible output for a given input. It is estimated in technical, allocative and overall aspects.

Keywords: Banking efficiency, Decision Making Units, Data Envelopment Analysis, Output Technical Efficiency, Super efficiency.

Introduction**Computation of output technical efficiencies**

Efficiency measurement was long back recognized as an important instrument in framing policies of organizations not necessarily profit making organizations. Farrell laid initial foundation for efficiency measurement which was later strengthened by Shephard, Charnes, Cooper and Rhodes, Banker Lovell and Fare and so on.

The approach of Farrell was empirical implicit in his assumptions the returns to scale were constant. Farrell identified three efficiency measurers viz., technical, allocative and overall efficiencies, where the over all (cost) efficiency is the product of technical and allocative efficiencies¹. A rigorous mathematical treatment was given to efficiency measurement by Shephard. Efficiency measurement can be measured by input and output distance functions.

To measure input technical efficiency, for a given level of output vector, we seek input reduction. For efficient decision making unit such input reduction is not possible. To measure input technical efficiency the input isoquants of input sets provide benchmark technology. If $L(u_0)$ is an input level set, Shephard defined it as,

$$L(u_0) = \{x: x \text{ produces } u_0\}$$

$$\text{Isoq } L(u_0) = \{x: x \in L(u_0), x' \leq x \Rightarrow x' \notin L(u_0)\}$$

Shephard's measure of input technical efficiency can be viewed as an optimization problem.

$$[D_i(u, x)]^{-1} = \text{Min } \{\lambda : \lambda x \in L(u)\} \\ = F_i(u, x)$$

where D_i and F_i are respectively the Shepherd's input distance function and Farrell's measure of input technical efficiency.

Traditionally, Indian commercial banks functioned constrained by the regulations which change from time to time as governed by the central bank, namely, the Reserve Bank of India. The objectives of the Indian commercial banks are

- Maximization of social benefit
- Minimization of risk
- Maximization of profit
- Expansion to reach more number of people of the country.

Charnes, Cooper and Rhodes (CCR)² formulated a fractional programming problem to measure output efficiency of decision making units.

¹ The production frontier assumed for efficiency measurement was linear homogeneous

$$\begin{aligned} \text{Minimize} \quad & \phi = \text{Min} \frac{\sum_{i=1}^m v_i X_{i0}}{\sum_{r=1}^s \mu_r u_{r0}} \\ \text{subject to} \quad & \frac{\sum_{i=1}^m v_i X_{ij}}{\sum_{r=1}^s \mu_r u_{rj}} \geq 1, j = 1, 2, \dots, n \quad v_i \geq 0, \mu_r \geq 0 \end{aligned}$$

Applying Charnes – Cooper transformation the fractional programming problem can be reduced into a linear programming problem.

$$\begin{aligned} \text{Min } \phi = \text{Min} \quad & \sum_{i=1}^m v_i X_{i0} \\ \text{subject to} \quad & \sum_{r=1}^s \mu_r u_{r0} = 1 \\ & \sum_{i=1}^m v_i X_{ij} - \sum_{r=1}^s \mu_r u_{rj} \geq 0, j = 1, 2, \dots, n \end{aligned} \quad \left. \vphantom{\sum_{i=1}^m v_i X_{i0}} \right\} v_i, \mu_r \geq 0 \text{ -----[1]}$$

n decision making units compete with each other while the frontier is determined by the best practice decision making units. For such decision making units,

$$\text{Min } \phi = 1$$

The multiplier problem (1) assume implicitly scale efficiency

Table. 1: Output Technical Efficiency

Bank	Over all output technical efficiency
Allahabad Bank	1.0074
Andhra Bank	1.0229
Bank of Baroda	1.0000
Bank of India	1.0000
Bank of Maharashtra	1.3387
Canara Bank	1.0000
Central Bank of India	1.0940
Corporation Bank	1.0000
Daha Bank	1.0000
IDBI Limited	1.0000
Indian Bank	1.0000
Indian Overseas Bank	1.0000
Oriental Bank	1.0000
Punjab & Sindh Bank	1.0000
Punjab National Bank	1.0000
Syndicate Bank	1.1402
UCO Bank	1.0000
Union Bank of India	1.0000
United Bank of India	1.0400
Vijaya Bank	1.0000

The dual of the above multiplier problem is again a linear programming problem, expressed as,

$$\begin{aligned} \theta^* = \text{Max } & \theta \\ \text{subject to} \quad & \sum_{j=1}^n \lambda_j X_{ij} \leq X_{i0} \quad i = 1, 2, \dots, m \end{aligned} \quad \left. \vphantom{\sum_{j=1}^n \lambda_j X_{ij}} \right\}$$

$$\sum_{j=1}^n \lambda_j u_{rj} \geq \theta u_{r0}, \quad r=1,2,\dots,s \quad \text{-----} \quad [2]$$

$$\lambda_j \geq 0$$

Each decision making unit combine ‘m’ inputs and produces outputs.

The optimization problem (1) is dualistically related. The extremities of their objective functions are equal.

$$\text{Min } \phi = \text{Max } \theta$$

In the optimal solution of the primal problem, if a primal variable is positive, its corresponding dual constraint is an equality so that the dual slack vanishes. On the other hand, if a primal variable is zero the corresponding dual constraint is a strict inequality so that the dual slack is positive.

After a careful examination of asset / liability variables the current study has identified two outputs, ‘interest income and other income’. These variables determine total income of a commercial bank. Four input variables are considered.

1. Input expended
2. Operating expenses
3. Deposits
4. Investments

Historically, there is a debate to consider deposits as input or output. Since a significant part of banks income is generated by lending and deposits serve as source of loanable funds, we choose deposits as input rather than output. A bank invests its capital in Govt. securities, mutual funds and so on and obtains income we prefer to choose ‘investments’ as input rather than output.

14 out of 20 commercial banks have emerged to be overall output technical efficient. None of these fourteen banks experienced any output losses attributed to technical inefficiency. Output losses are experienced by Andhra Bank and United Bank of India only marginally. Had the Bank of Maharashtra been overall output technical efficient, it could have produced 34 per cent more output than it produces currently. The Central Bank of India suffers from 9.4 per cent of output losses. Had the Syndicate Bank been overall output technical efficient it could have produced 14 per cent more output than it produces currently.

The efficiency estimates are relative, the bench marks being provided by the best practicing commercial banks. If a bank is added to these 20 and this bank used smaller inputs and produced large outputs than the efficient banks among 20 then new bench marks will arise. Consequently, one or more of the previous efficient banks will emerge to be inefficient.

Every inefficient bank is governed by a peer group each of whose member is an extremely efficient bank. If bank ‘j’ is extremely efficient, in the optimal solution of the envelopment problem solved for jth bank we obtain,

$$\lambda_k^* = 1, \quad k = j$$

$$= 0, \quad k \neq j$$

However, if bank ‘j’ is not efficient, then we have,

$$\lambda_k^* = 0, \quad k = j$$

$$\neq 0 \quad \text{for one or more } k \neq j$$

The 20 Commercial Banks analyzed, can be grouped into four categories, (i) extremely efficient, (ii) efficient but not extremely efficient, (iii) weakly efficient and (iv) inefficient

For Allahabad Bank, the Bank of Baroda, UCO Bank and Union Bank of India together represent the peer group. The inefficient bank is similar in characteristics such as size, management and other environmental variables to the extremely efficient DMUs which represent its peer list.

In terms of spread management Bank of Baroda and Union Bank of India appear to be better than the inefficient Allahabad Bank. Andhra Bank which emerged to be overall output technical inefficient has the peer list consisting of Corporation Bank, Indian Bank, Indian Overseas Bank and Punjab and Sindh Bank.

The Bank of Maharashtra appears to be the most inefficient losing a significant portion of potential outputs due to over all output technical inefficiency. The Corporation Bank, Indian Overseas Bank, Punjab and Sindh Bank and Union Bank of India form the peer group of the Bank of Maharashtra. When measured output technical efficiency the Central Bank of India can be placed just above the Bank of Maharashtra. Its peer list members are Indian Overseas Bank and Punjab National Bank.

Syndicate Bank lost 14 per cent of its potential outputs and its peer list consists of the following extremely efficient decision making units.

1. Corporation Bank
2. Indian Overseas Bank

3. Punjab and Sindh Bank and
4. Union Bank of India

Imitating the practices of the above four extremely efficient commercial banks, Syndicate Bank may reduce output losses, consequently attaining over all output technical efficiency.

The United Bank of India also suffered from output losses, but only marginally. Its peer list consists of Corporation Bank, Indian Bank and Indian Overseas Bank.

Vijaya Bank is output technical efficient. It is efficient but, not extremely efficient. The Corporation Bank, Indian Overseas Bank and Union Bank of India are its peers.

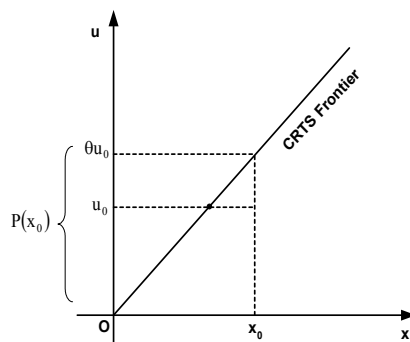


Fig. 1. Production process that employs one input and produces one output

The figure above narrates a production process that employs one input and produces one output. The straight line that emanates from the origin is the frontier production function that admits only constant returns to scale.

Table. 2. Peers and Peer weights of inefficient commercial banks

Inefficient DMU	Peers and their weights
Allahabad Bank	: UCO Bank (0.041), Punjab National Bank (0.282), Oriental Bank (0.852), Union Bank of India (0.325)
Andhra Bank	: Corporation Bank (0.062), Indian Overseas Bank (0.105), Bank of Baroda (0.053), Indian Bank, (0.269), Punjab National Bank (0.511)
Bank of Maharashtra	: Bank of Baroda (0.019), Indian Overseas Bank (0.124), Punjab and Sindh Bank (0.677), Corporation Bank (0.677), Union Bank of India (0.039)
Central Bank of India	: Indian Overseas Bank (0.773), Bank of Baroda (0.227)
Syndicate Bank	: Indian Bank (0.127), Bank of Baroda (0.024), Indian Overseas Bank (0.120), Union Bank of India (0.616), Corporation Bank (0.113)
United Bank of India	: Corporation Bank (0.151), Indian Overseas Bank (0.214), Indian Bank (0.037), Punjab National Bank (0.594)

The names of the commercial banks displayed against each inefficient bank are its peers. The numbers in parentheses are the intensity parameters assigned to each extremely efficient commercial bank in the peer list.

The r^{th} output of Allahabad Bank may be expressed as

$$\sum_{j=1}^n \lambda_j u_{rj} - s_j^+ = \theta u_{r0}, \quad r = 1, 2, \dots, s$$

If $\lambda_j^*, (s_j^+)^*, \theta^*$ stand for optimal solution of Allahabad Bank

$$\sum_{j=1}^n \lambda_j^* u_{rj} - (s_j^+)^* = \theta^* u_{r0}, \quad r = 1, 2, \dots, s$$

where $\lambda_{13}^* = 0.352 \quad \lambda_{15}^* = 0.282 \quad \lambda_{17}^* = 0.041 \quad \lambda_{18}^* = 0.325$

All other intensity parameters vanish $\theta^* = 0.993$

Input slacks : $(s_3^-)^* = 1253.745$

Output slacks : $(s_2^+)^* = 139.044$. All other slacks vanished

The above peer list is obtained solving output DEA imposing constant returns to scale. Often economic data are subjected to returns to scale. For a commercial bank returns to scale are constant, or increasing, or decreasing. Any DMU that enjoys constant returns to scale is scale efficient. To identify returns to scale, we augment a constant term to the objective function of the multiplier problem.

$$\begin{aligned} \text{Minimize} \quad & \phi = \sum_{i=1}^m v_i x_{i0} + \delta \\ \text{subject to} \quad & r \sum_{r=1}^s \mu_r u_{r0} = 1 \\ & \sum_{i=1}^m v_i x_{ij} - \sum_{r=1}^s \mu_r u_{rj} + \delta \geq 0 \quad v_i, \mu_r \geq 0 \end{aligned}$$

In the optimal solution,

- $\delta^* = 0 \Rightarrow$ Constant returns to scale
- $\delta^* > 0 \Rightarrow$ Increasing returns to scale
- $\delta^* < 0 \Rightarrow$ Decreasing returns to scale

The dual of the above problem is the output envelopment problem

$$\begin{aligned} \text{Max } \theta \quad & \text{subject to} \\ & \sum_{j=1}^n \lambda_j x_{rj} \leq x_{r0} \quad r = 1, 2, \dots, m \\ & \sum_{j=1}^n \lambda_j u_{rj} \geq \theta u_{r0} \quad r = 1, 2, \dots, s \\ & \sum_{j=1}^n \lambda_j = 1 \end{aligned}$$

$$P^v(x_0) \leq P^K(x_0)$$

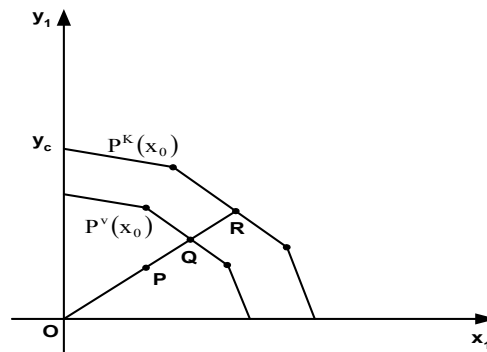


Fig. 2. The DMU that operates at ‘P’ is output technical inefficient. $P^v(x_0)$ and $P^K(x_0)$ are the output level sets admitting variable and constant returns to scale respectively.

Output technical efficiency under constant returns to scale

$$\theta^K = \frac{OR}{OP}$$

Output technical efficiency under increasing or decreasing returns to scale

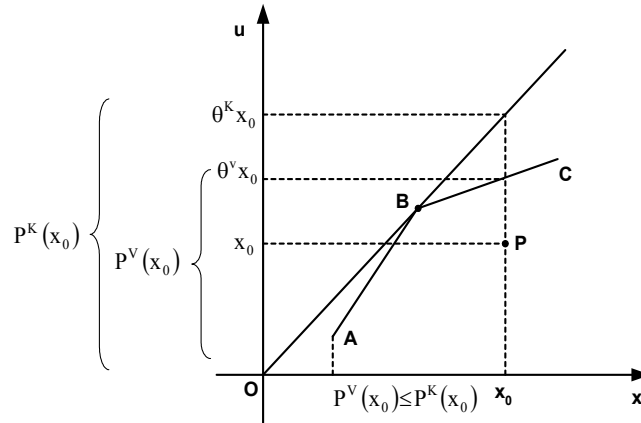
$$\theta^v = \frac{OQ}{OP}, \quad \theta^K \geq \theta^v$$

Output scale efficiency is the ratio of θ^K to θ^v

$$OSE = \frac{\theta^K}{\theta^v}$$

Thus, we have

$$\theta^K = (OSE)\theta^v$$



If a decision making unit is output overall technical efficient it is output pure technical efficient also. But, the converse is untrue.

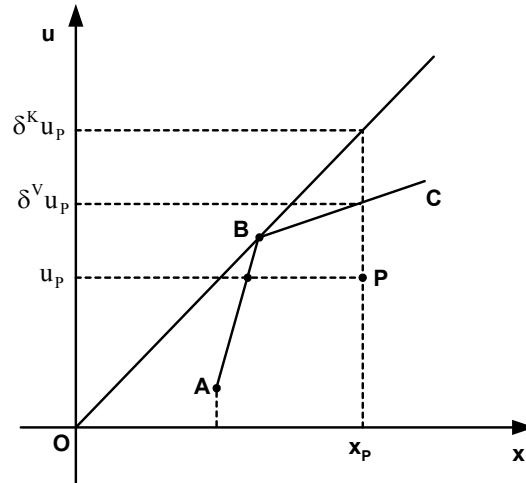
Table 3. The following table furnishes output overall, pure and scale efficiencies of 20 commercial banks.

S. No.	Bank Name	CRTS	VRTS	SCALE
1.	Allahabad Bank	1.0173	1.0070	0.0101
2.	Andhra Bank	1.0142	1.0101	1.0040
3.	Bank of Baroda	1.000	1.0000	1.0000
4.	Bank of India	1.000	1.0000	1.0000
5.	Bank of Maharashtra	1.2516	1.2516	1.0000
6.	Canara Bank	1.0225	1.0000	1.0225
7.	Central Bank of India	1.0776	1.0582	1.0173
8.	Corporation Bank	1.0000	1.0000	1.0000
9.	Dena Bank	1.0000	1.0000	1.0000
10.	IDBI Ltd	1.0000	1.0000	1.0000
11.	Indian Bank	1.0000	1.0000	1.0000
12.	Indian Overseas Bank	1.0000	1.0000	1.0000
13.	Oriental Bank	1.0000	1.0000	1.0000
14.	Punjab & Sindh Bank	1.0000	1.0000	1.0000
15.	Punjab International Bank	1.0000	1.0000	1.0000
16.	Syndicate Bank	1.0638	1.0593	1.0040
17.	UCO Bank	1.0000	1.0000	1.0000
18.	Union Bank of India	1.0000	1.0000	1.0000
19.	United Bank of India	1.0449	1.0373	1.0070
20.	Vijaya Bank	1.0438	1.0000	1.0438

Canara Bank and Vijaya Bank which were technically inefficient under constant returns to scale hypothesis emerged to be pure technical efficient. Allahabad Bank, Andhra Bank, Bank of Maharashtra, Canara Bank, Central Bank of India, Syndicate Bank, United Bank of India, and Vijaya Bank have experienced output losses due to scale inefficiency.

Allahabad Bank, Andhra Bank, Bank of Maharashtra, Central Bank of India, Syndicate Bank, and United Bank of India are found output pure technical inefficient. However, output losses experienced by these commercial banks are only marginal, except the Bank of Maharashtra. Had the Bank of Maharashtra been output pure technical efficient it would have produced 25 per cent more outputs than it currently produces without changing its inputs.

2. Identification of returns to scale



The ray that emanates from the origin is constant returns to scale frontier. The area bound by CRTS frontier is CRTS production possibility set.

$$\left\{ (x, u) : \sum_{j=1}^n \lambda_j x_{ij} \leq x_i, \sum_{j=1}^n \lambda_j u_{rj} \geq u_r, \lambda_j \geq 0 \right\}$$

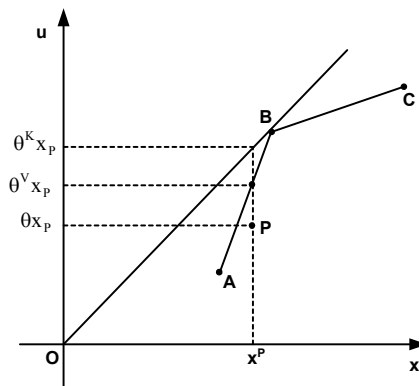
The line segments OB and BC constitute non-increasing returns to scale frontier and the production possibility set bound by it is,

$$\left\{ (x, u) : \sum_{j=1}^n \lambda_j x_{ij} \leq x_i, \sum_{j=1}^n \lambda_j u_{rj} \geq u_r, \sum_{j=1}^n \lambda_j \leq 1 \right\}$$

The producer who operates at ‘P’ is output inefficient, for whom we solve three linear programming problems:

1. $\theta^K = \text{Max} \left\{ \theta : \sum_{j=1}^n \lambda_j x_{ij} \leq x_{i0}, \sum_{j=1}^n \lambda_j u_{rj} \geq \theta u_{r0}, \right\}$
2. $\theta^V = \text{Max} \left\{ \theta : \sum_{j=1}^n \lambda_j x_{ij} \leq x_{i0}, \sum_{j=1}^n \lambda_j u_{rj} \geq \theta u_{r0}, \sum_{j=1}^n \lambda_j = 1 \right\}$
3. $\theta^{NI} = \text{Max} \left\{ \theta : \sum_{j=1}^n \lambda_j x_{ij} \leq x_{i0}, \sum_{j=1}^n \lambda_j u_{rj} \geq \theta u_{r0}, \sum_{j=1}^n \lambda_j \leq 1 \right\}$

$\theta^{NI} = \theta^V < \theta^K \Rightarrow$ Returns to scale are decreasing. For decision making unit ‘P’ output based returns to scale are decreasing.



$\theta^{NI} = \theta^K > \theta^V \Rightarrow$ Returns to scale are increasing

$\theta^V = \theta^K \Rightarrow$ Returns to scale are constant

1. $(\theta^K)^{-1} < (\theta^V)^{-1} = (\theta^{NI})^{-1} \Rightarrow$ Returns to scale are decreasing

- 2. $(\theta^{NI})^{-1} = (\theta^K)^{-1} < (\theta^V)^{-1} \Rightarrow$ Returns to scale are increasing
- 3. $(\theta^V)^{-1} = (\theta^K)^{-1} \Rightarrow$ Returns to scale are constant

The table (4.2.1) portrays the inverses of output technical efficiencies.

The decision making unit P is output technical inefficient. For this DMU we have,

$$(\theta^{NI})^{-1} = (\theta^K)^{-1} < (\theta^V)^{-1}$$

Table.4. Returns to scale of ‘P’ are increasing.

S.No.	Commercial Bank	Returns to Scale
1.	Allahabad Bank	IRS
2.	Andhra Bank	DRS
3.	Bank of Baroda	CRS
4.	Bank of India	CRS
5.	Bank of Maharashtra	CRS
6.	Canara Bank	DRS
7.	Central Bank of India	DRS
8.	Corporation Bank	CRS
9.	Dena Bank	CRS
10.	IDBI Ltd	CRS
11.	Indian Bank	CRS
12.	India Overseas Bank	CRS
13.	Oriental Bank	CRS
14.	Punjab and Sindh Bank	CRS
15.	Punjab National Bank	CRS
16.	Syndicate Bank	DRS
17.	UCO Bank	CRS
18.	Union Bank of India	CRS
19.	United Bank of India	IRS
20.	Vijaya Bank	IRS

Allahabad Bank, United Bank of India and Vijaya Bank enjoy increasing returns to scale (IRS). Andhra Bank, Canara Bank, Central Bank of India, and Syndicate Bank, suffer from decreasing returns to scale. All other commercial banks are scale efficient, enjoy constant returns to scale.

For each decision making unit we enquire for possible output expansion. If further output expansion is possible, the DMU is output technical inefficient, otherwise efficient. If a DMU is inefficient projected values of outputs of the DMU need to be calculated.

ALLAHABAD BANK:

Original Output (1)	:	4884
Original Output (2)	:	376
Projected Output (1)	:	4919.985
Projected Output (2)	:	517.815

ANDHRA BANK:

Original Output (1)	:	3315
Original Output (2)	:	447
Projected Output (1)	:	3348.561
Projected Output (2)	:	451.525

BANK OF MAHARASHTRA:

Original Output (1)	:	2272
Original Output (2)	:	265
Projected Output (1)	:	2840.638
Projected Output (2)	:	331.324

CENTRAL BANK OF INDIA:

Original Output (1)	:	6234
Original Output (2)	:	476
Projected Output (1)	:	6598.841
Projected Output (2)	:	565.272

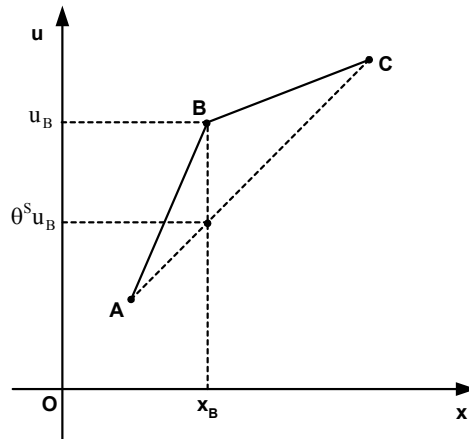
SYNDICATE BANK:

Original Output (1)	:	6040
Original Output (2)	:	618
Projected Output (1)	:	6400.275

Projected Output (2)	:	654.863
UNITED BANK OF INDIA:		
Original Output (1)	:	2853
Original Output (2)	:	320
Projected Output (1)	:	2960.375
Projected Output (2)	:	332.043

Super Efficiency

In the presence of 100 per cent efficiency score DMUs whose number exceeds one DEA loses its discriminatory power, in the sense that it fails to rank the efficient DMUs. This problem, may possibly be resolved if there are super efficient DMUs among the DMUs whose efficiency scores are 100 per cent.



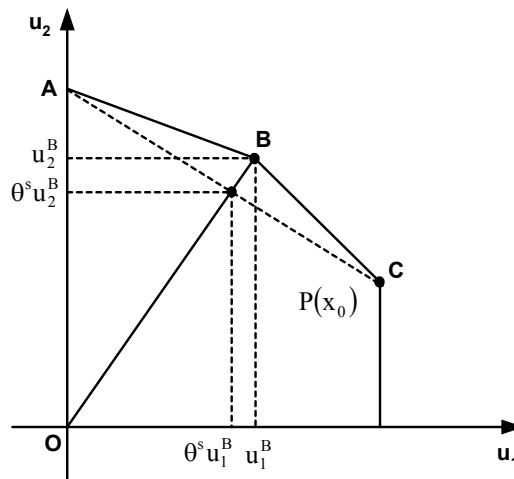
DMUs A, B and C are extremely efficient. To compute output super efficiency of DMU B, we remove the input and output vectors of DMU B from the reference technology. Consequently, we represent the input and output of the composite DMU by,

$$\left(\sum_{\substack{j=1 \\ j \neq B}}^n \lambda_j x_{ij}, \sum_{\substack{j=1 \\ j \neq B}}^n \lambda_j u_{rj}, \sum_{\substack{j=1 \\ j \neq B}}^n \lambda_j = 1, \lambda_j \geq 0 \right)$$

The piecewise linear production frontier consists of the segment AC and it does not contain the segments AB and BC. Super efficiency scores are computed only to extremely efficient and efficient but not extremely efficient DMUs. The producer who operates at 'B' employs input 'x_B' and produces output 'u_B'. The referent DMUs of 'B' are DMUs A and C.

The super efficiency problem projects 'u_B' vertically onto the super efficient frontier and 'u_B' is contracted from its current level to θ^s u_B. θ^s < 1

Employing the input 'u_B' the DMUs A and B would have jointly produced θ^s u_B < u_B. Thus, DMU B is super efficient, produces an additional output (1-θ^s) u_B.



$P(x_0)$ is the output level set and x_0 is scalar input. If DMU B is removed from reference technology and the super efficiency problem is solved, both the outputs are reduced the first one from u_1^B to $\theta^s u_1^B$ and the second one from u_2^B to $\theta^s u_2^B$ (radial reduction). The additional outputs produced by DMU B due to its super efficiency are $(1-\theta^s) u_1^B$ and $(1-\theta^s) u_2^B$.

Table.5. Output Super Efficiency

Efficient DMU	Super Efficiency	Rank
Bank of Baroda	99.00	11
Bank of India	92.27	3
Corporation Bank	93.71	5
Dena Bank	92.37	4
IDBI Bank	52.92	1
Indian Bank	91.29	2
Indian overseas Bank	97.86	10
Oriental Bank	99.39	12
Punjab and Sindh Bank	94.92	7
Punjab National Bank	93.92	6
UCO Bank	96.86	9
Union Bank of India	96.67	8

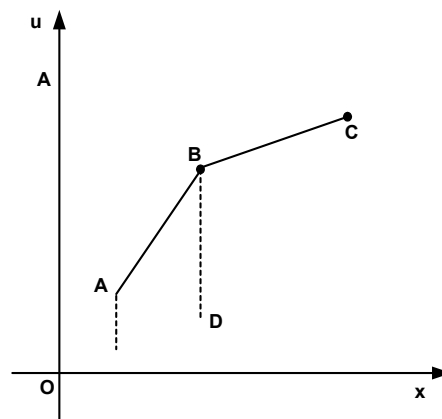
There are 12 extremely efficient decision making units and each one of them is output super efficient. In terms of additional outputs produced IDBI bank stands in the first place the super efficient decision making units are assigned with ranks ranging from one to twelve.

Table 7. Output inefficiency

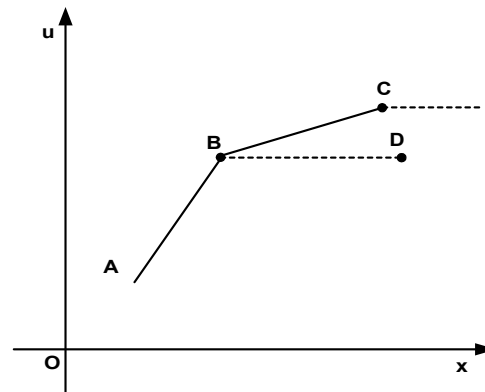
Inefficient DMU	Efficiency	Rank
Allahabad Bank	101.77	13
Andhra Bank	102.66	15
Bank of Maharashtra	126.11	20
Central Bank	102.20	14
Central Bank of India	108.75	19
Syndicate Bank	106.52	18
United Bank of India	105.37	17
Vijaya Bank	104.34	16

Ranks of inefficient DMUs varied between 13 and 20. Allahabad Bank, Andhra Bank and Central Bank have suffered output losses marginally. The Bank of Maharashtra suffered maximum output loss, secured bottom most rank viz., 20. Had this bank been output technical efficient it could have produced 26 per cent more of the outputs than it produces currently without changing input levels.

Super Efficiency – Infeasibility



Consider figure in which DMU A is removed from the reference technology. The super efficiency frontier consists the line segments BC and BD. If the super efficiency problem is solved for DMU A, no feasible solution exists and the DMU is classified as “BIG”.



If C is removed from the reference technology, the super efficiency frontier is constituted by the line segments AB and BD. If the output super efficiency problem is solved for DMU C as there is no referent DMU, no feasible solution exists. Consequently DMU C is assigned to category “BIG”. The DMUs classified big remained to be efficient for infinite reduction of inputs and augmentation of outputs.

Peer Count

An inefficient commercial bank possesses a peer list. The peer members are the inefficient DMU's role models. An efficient commercial bank is a role model to itself. The efficient commercial banks constitute the benchmark technology. Allahabad Bank experienced output losses due to inefficiency. Among its peer banks, Oriental Bank is the most influential bank and its practices are the best practices of Allahabad Bank.

Another inefficient commercial bank is Andhra Bank whose most influential peer bank is Punjab National Bank whose intensity parameter value is 0.511. To gain output technical efficiency Andhra Bank shall adjust its performance indicators in tune with its peers particularly Punjab National Bank.

Bank of Maharashtra is yet another inefficient commercial bank, dominated by Punjab and Sindh Bank and Corporation Bank with equal intensity measured by the intensity parameter value 0.677.

Central Bank of India has only two peers in its list of which Indian Overseas Bank emerge to be more influential than Bank of Baroda. The practices of Indian Overseas Bank are the best practices of Central Bank of India.

Syndicate Bank has five efficient banks in its peer list of which Union Bank of India is the most influential commercial bank with its intensity parameter assuming the value 0.616.

Union Bank of India consists of four extremely efficient DMUs in its peer list. Punjab National Bank is the most dominating peer of Union Bank of India.

Maximum peer count is experienced by Indian Overseas Bank, the count being 5 Banks of Baroda and Corporation Bank were in the peer lists of four inefficient commercial banks each.

Indian Bank, Punjab National Bank and Union Bank of India are in the peer lists of three commercial banks each, whereas Oriental Bank, Punjab and Sindh Bank and UCO bank are found in the peer lists of one inefficient bank each. Every commercial is peer of itself.

To rank efficient decision making units some times peer count also is used, choosing the DMU with largest peer count as the best one. However, efficient DMUs emerge tied with respect to their peer count again leading to loss of discriminating power.

Conclusion

The present study primarily classifies commercial banks as efficient and inefficient and for inefficient banks it provides 'role models' that are always extremely efficient. Often economic data are constrained by returns to scale and this study identified for each commercial bank the sources of returns to scale as constant or increasing or decreasing. 12 out of 20 banks are found to be scale efficient and they do not experience output losses due to scale inefficiency. The other commercial banks are nearly scale efficient; suffer from output losses only marginally due to output scale inefficiency.

For each commercial bank overall output technical efficiency is computed and decomposed it into the product of output pure technical and scale efficiencies.

$$OOTE = OPTE \times OSE$$

Where; OOTE : Overall Output technical efficiency

Computed ignoring the constraint

$$\sum_{j=1}^n \lambda_j = 1$$

OPTE : Output pure technical efficiency

OSE : Output scale efficiency

Output scale efficiency is a derived measure obtained as the following ratio.

$$OSE = \frac{OOTE}{OPTE}$$

The scale efficient banks are Bank of Baroda, Bank of India, Bank of Maharashtra, Corporation Bank, Dena Bank, IDBI Ltd, Indian Bank, Indian Oversea Bank, Oriental Bank, Punjab & Sindh Bank, Punjab International Bank, UCO Bank, and Union Bank of India. 14 out of 20 banks are output pure technical efficient. To estimate output pure technical efficiency, the following linear programming problem is solved.

The commercial banks rated output pure technical efficient are, Bank of Baroda, Bank of India, Canara Bank, Corporation Bank, Dena Bank, IDBI Ltd, Indian Bank, Indian Overseas Bank, Oriental Bank, Punjab & Sindh Bank, Punjab International Bank, UCO Bank, Union Bank of India and Vijaya Bank. Of these fourteen banks there is only one commercial bank namely canara bank that is not overall output technical efficient.

In the presence of several DMUs rated efficient, DEA fails to distinguish these decision making units. DEA shall not only measure efficiency but also rank the commercial Banks. We find 12 commercial banks that are overall output technical efficient. To rank these banks we computed output super efficiency for each of them.

Output super efficiency is computed to extremely efficient decision making units. Classical and super efficiency remain to be the same for inefficient decision making units. To compute super efficiency (output) the following linear programming problem is solved for each commercial bank.

The super efficiency of IDBI bank is 52.92 per cent. If the inputs employed by IDBI bank are given to other commercial banks, collectively they could have produced 52.92 per cent of the output of IDBI bank. Output gain experienced by IDBI bank due to super efficiency is 47 per cent. Smaller super efficiency score implies greater efficiency.

Table 6. Output efficiency/ super efficiency scores

Commercial Bank	Super Efficiency / Efficiency	Rank
Bank of Baroda	99.00	11
Bank of India	92.27	3
Corporation Bank	93.71	5
Dena Bank	92.37	4
IDBI Bank	52.92	1
Indian Bank	91.29	2
Indian overseas Bank	97.86	10
Oriental Bank	99.39	12
Punjab and Sindh Bank	94.92	7
Punjab National Bank	93.92	6
UCO Bank	96.86	9
Union Bank of India	96.67	8
Allahabad Bank	101.77	13
Andhra Bank	102.66	15
Bank of Maharashtra	126.11	20
Central Bank	102.20	14
Central Bank of India	108.75	19
Syndicate Bank	106.52	18
United Bank of India	105.37	17
Vijaya Bank	104.34	16

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