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Full Length Research Paper

Velocity and Acceleration in Rotational Prolate Spheroidal Coordinates

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Abstract
In this paper, we formulate the instantaneous velocity and acceleration vector in Rotational Prolate Spheroidal Coordinates to expand the scope of mechanics for any body in rotational prolate spheroidal coordinates.

Key words: *Rotational Prolate Spheroidal Coordinates, Instantaneous Velocity and Instantaneous Acceleration Vectors.*

Introduction

We had formulated the instantaneous velocity and acceleration in oblate spheroidal, prolate spheroidal, parabolic cylindrical, parabolic and second torridal coordinates in addition to the well-known instantaneous velocity and acceleration in Cartesian, cylindrical and polar spherical coordinates [1, 2, 3, 4, 5]. In this paper, we are out again to establish the instantaneous velocity and instantaneous acceleration in rotational prolate spheroidal coordinates to expand the scope of mechanics in physics and mathematics.

The Rotational Prolate Spheroidal Coordinates (u, v, w) are related to the Cartesian coordinates (x, y, z) as [6]:

$$x = w(u^2 - d^2)^{\frac{1}{2}}(1 - v^2)^{\frac{1}{2}} \tag{1}$$

$$y = (u^2 - d^2)^{\frac{1}{2}}(1 - v^2)^{\frac{1}{2}}(1 - w^2)^{\frac{1}{2}} \tag{2}$$

$$y = uv \tag{3}$$

Consequently, by definition, the Rotational Prolate Spheroidal Metric Coefficients are given by:

$$h_u = \frac{(u^2 - v^2 d^2)^{\frac{1}{2}}}{(u^2 - d^2)^{\frac{1}{2}}} \tag{4}$$

$$h_v = \frac{(u^2 - v^2 d^2)^{\frac{1}{2}}}{(1 - v^2)^{\frac{1}{2}}} \tag{5}$$

$$h_w = \frac{(u^2 - d^2)^{\frac{1}{2}}(1 - v^2)^{\frac{1}{2}}}{(1 - w^2)^{\frac{1}{2}}} \tag{6}$$

The Metric Coefficient define unit vectors line element, volume element, gradient, divergence, curl and Laplacican Operations in Rotational Prolate Spheroidal Coordinates according to the theory of orthogonal curvilinear coordinates [7, 8, 9].

These quantities are necessary and sufficient for the derivation of the fields of all Rotational Prolate Spheroidal distribution of mass, charge and current. Therefore for the derivation of the equation of motion for test particles in these fields, we shall derive the expression for instantaneous velocity and acceleration in Rotational Prolate Spheroidal Coordinates.

Mathematical Analysis

The Cartesian unit vectors are related to the Rotational Prolate Spheroidal Coordinates unit vectors as:

$$\hat{u} = \frac{uw(1-v^2)^{\frac{1}{2}}}{(u^2-v^2d^2)^{\frac{1}{2}}}\hat{\imath} + \frac{u(1-v^2)^{\frac{1}{2}}(1-w^2)^{\frac{1}{2}}}{(u^2-v^2d^2)^{\frac{1}{2}}}\hat{\jmath} + \frac{v(u^2-d^2)^{\frac{1}{2}}}{(u^2-u^2d^2)^{\frac{1}{2}}}\hat{k} \quad (7)$$

$$\hat{u} = \frac{-vw(1-v^2)^{\frac{1}{2}}}{(u^2-v^2d^2)^{\frac{1}{2}}}\hat{\imath} - \frac{v(u^2-d^2)^{\frac{1}{2}}(1-w^2)^{\frac{1}{2}}}{(u^2-v^2d^2)^{\frac{1}{2}}}\hat{\jmath} + \frac{u(1-v^2)^{\frac{1}{2}}}{(u^2-u^2d^2)^{\frac{1}{2}}}\hat{k} \quad (8)$$

$$\hat{w} = (1-v^2)^{\frac{1}{2}}\hat{\imath} - w\hat{\jmath} \quad (9)$$

The inversion is given as;

$$\hat{\imath} = \frac{uw(1-v^2)^{\frac{1}{2}}}{(u^2-v^2d^2)}\hat{u} - \frac{(u^2-d^2)^{\frac{1}{2}}}{(u^2-v^2d^2)^{\frac{1}{2}}}\hat{v} + (1-w^2)^{\frac{1}{2}}\hat{w} \quad (10)$$

$$\hat{\jmath} = \frac{u(1-v^2)^{\frac{1}{2}}(1-w^2)^{\frac{1}{2}}}{(u^2-v^2d^2)}\hat{u} - \frac{v(u^2-d^2)^{\frac{1}{2}}(1-w^2)^{\frac{1}{2}}}{(u^2-v^2d^2)^{\frac{1}{2}}}\hat{v} - w\hat{w} \quad (11)$$

$$\hat{k} = \frac{v(u^2-d^2)^{\frac{1}{2}}}{(u^2-v^2d^2)}\hat{u} + \frac{u(1-v^2)^{\frac{1}{2}}}{(u^2-v^2d^2)^{\frac{1}{2}}}\hat{v} \quad (12)$$

Hence denoting one time differentiation by a dot, it follows from (7), (8), and (9) and some manipulation that:

$$\dot{\hat{u}} = \frac{1}{(u^2-v^2d^2)}\left[\frac{vd^2(1-v^2)^{\frac{1}{2}}}{(u^2-d^2)^{\frac{1}{2}}}\dot{u} + \frac{u(u^2-d^2)^{\frac{1}{2}}}{(1-v^2)^{\frac{1}{2}}}\dot{v}\right]\hat{v}$$

$$+ \frac{u(1-v^2)^{\frac{1}{2}}}{(1-w^2)^{\frac{1}{2}}(u^2-d^2)^{\frac{1}{2}}}\dot{w}\hat{w} \quad (13)$$

Similarly, it follows from (8), (7), and (9) that:

$$\dot{\hat{v}} = \frac{-1}{(u^2-v^2d^2)}\left[\frac{vd^2(1-v^2)^{\frac{1}{2}}}{(u^2-d^2)^{\frac{1}{2}}}\dot{u} + \frac{u(u^2-d^2)^{\frac{1}{2}}}{(1-v^2)^{\frac{1}{2}}}\dot{v}\right]\hat{u}$$

$$- \frac{v(u^2-d^2)^{\frac{1}{2}}}{(1-w^2)^{\frac{1}{2}}(u^2-v^2d^2)^{\frac{1}{2}}}\dot{w}\hat{w} \quad (14)$$

And consequently from (9), (10) and (11);

$$\dot{\hat{w}} = \frac{1}{(1-w^2)^{\frac{1}{2}}(u^2-v^2d^2)^{\frac{1}{2}}}\left[-u(1-v^2)^{\frac{1}{2}}u + v(u^2-d^2)^{\frac{1}{2}}\hat{v}\right]\dot{w} \quad (15)$$

Now it follows from definition of instantaneous position vector $\underline{r}$ as

$$\underline{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k} \quad (16)$$

And from (1) – (3) and (10) – (12) that the instantaneous position vector may be expressed entirely in terms of Rotational Prolate Spheroidal Coordinates as:

$$\underline{r} = \frac{u(u^2-d^2)^{\frac{1}{2}}}{(u^2-v^2d^2)^{\frac{1}{2}}}\hat{u} + \frac{vd^2(1-v^2)^{\frac{1}{2}}}{(u^2-v^2d^2)^{\frac{1}{2}}}\hat{v} \quad (17)$$

It now follows from definition of instantaneous velocity vector $\underline{u}$ as:

$$\underline{u} = \underline{\dot{r}} \quad (18)$$

And (17), (13) and (12) that the instantaneous velocity vector may be expressed entirely in term of Rotational Prolate Spheroidal Coordinates as:

$$\underline{u} = u_u \hat{u} + u_v \hat{v} + u_w \hat{w} \quad 19)$$

where;

$$u_u = \left(\frac{u^2 - v^2 d^2}{u^2 - d^2}\right)^{\frac{1}{2}} \dot{u} \quad (20)$$

$$u_v = \left(\frac{u^2 - v^2 d^2}{1 - v^2}\right)^{\frac{1}{2}} \dot{v} \quad (21)$$

$$u_w = \left(\frac{(u^2 - d^2)(1 - v^2)}{1 - w^2}\right)^{\frac{1}{2}} \dot{w} \quad (22)$$

Similarly, it follows from definition of instantaneous acceleration vector, $\underline{a}$ as;

$$\underline{a} = \underline{\dot{u}} \quad (23)$$

And (19), (13)-(15) that the instantaneous acceleration may be expressed entirely in term of Rotational Prolate Spheroidal coordinate as:

$$\underline{a} = a_u \hat{u} + a_v \hat{v} + a_w \hat{w} \quad (24)$$

Where;

$$a_u = \frac{(u^2 - v^2 d^2)^{\frac{1}{2}}}{(u^2 - d^2)^{\frac{1}{2}}}\left[\ddot{u} + \frac{u^2 d^2 (1 - v^2)}{(u^2 - d^2)(u^2 - v^2 d^2)}\dot{u}^2 - \frac{2 v d^2}{(u^2 - v^2 d^2)}\dot{u}\dot{v} - \frac{u(u^2 - d^2)}{(1 - v^2)(u^2 - u^2 d^2)}\dot{v}^2 - \frac{u(1 - v^2)(u^2 - d^2)}{(1 - w^2)(u^2 - v^2 d^2)}\dot{w}^2\right] \quad (25)$$

$$a_v = \frac{(u^2 - v^2 d^2)^{\frac{1}{2}}}{(1 - v^2)^{\frac{1}{2}}}\left[\ddot{v} + \frac{v(u^2 - d^2)}{(1 - v^2)(u^2 - v^2 d^2)}\dot{v}^2 + \frac{2u}{(u^2 - v^2 d^2)}\dot{u}\dot{v} - \frac{v d^2 (1 - v^2)}{(u^2 - d^2)(u^2 - u^2 d^2)}\dot{u}^2 - \frac{v(1 - v^2)(u^2 - d^2)}{(1 - w^2)(u^2 - v^2 d^2)}\dot{w}^2\right] \quad (26)$$

$$a_w = \frac{(u^2 - d^2)^{\frac{1}{2}}(1 - v^2)^{\frac{1}{2}}}{(1 - w^2)^{\frac{1}{2}}}\left[\ddot{w} + \frac{w}{1 - w^2}\dot{w}^2 - \frac{2v}{(1 - v^2)}\dot{v}\dot{w} - \frac{2u}{(u^2 - d^2)}\dot{u}\dot{w}\right] \quad (27)$$

This is the completion of the instantaneous velocity and the instantaneous acceleration in Rotational Prolate Spheroidal Coordinates system.

Result and Discussion

In this paper we derived the component of velocity and acceleration in Rotational Prolate Spheroidal Coordinates as (19) – (22) and (24) – (27), and are necessary and sufficient for expressing all mechanical quantities (linear momentum, kinetic energy, Lagrangian and Hamiltonian) in terms of Rotational Prolate Spheroidal Coordinates.

Conclusion

The velocity and acceleration component (20), (21), (22), (25), (26), (27) obtained in this paper paves a way for expressing all dynamical laws of motion (Newton Law, Lagrange's Law, Hamilton's Law, Einstein's Special relativities law of motion and Schrodinger's law of quantum mechanics) entirely in terms of Rotational Prolate Spheroidal Coordinates.

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