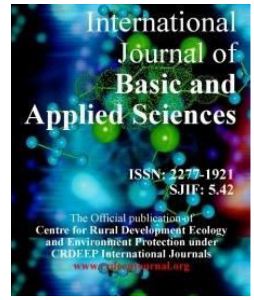
Vol. 8. No. 1. 2019

Contents available at:

www.crdeepjournal.org

International Journal of Basic and Applied Sciences (ISSN: 2277-1921)



Full Length Research Paper

On Edge- Antimagic Total Labeling of Dually Connected Graphs

H. U. Afzal

Department of Mathematics, GC University, Lahore-54000, Pakistan.

ARTICLE INFORMATION

Corresponding Author:
H. U Afzal

Article history:
Received: 01-12-2018
Revised: 10-12-2018
Accepted: 10-01-2019
Published: 29-01-2019

Key words:
Edge-antimagic total labeling, dual graphs, connected graphs.

ABSTRACT

An (a, d) edge-antimagic total labeling of a graph G is a bijection $\rho: V(G) \cup E(G) \rightarrow \{1, 2, \ldots, p + q\}$ such that the set of edge-weights of all edges in G, $\{w(xy) = \rho(x) + \rho(xy) + \rho(y) : xy \in E(G)\}$, forms an arithmetic progression $\{a, a+d, a+2d, \ldots, a+(q-1)d\}$, where $a > 0$ and $d \geq 0$ are fixed integers. We prove here that our dually connected graphs admit such progression by computing at least one such labeling for all of them, and rest are open to be found for others.

AMS 2010 MATHEMATICS SUBJECT CLASSIFICATION: *05C78, 05C80*

Introduction.

For a graph simple graph G, $V(G)$ and $E(G)$ denote the vertex set and the edge set, respectively. A (p, q)-graph G is one with $|V(G)| = p$ and $|E(G)| = q$. Moreover, the theoretic ideas of graphs can be seen in [21]. A labeling (or valuation) of a graph is in fact a mapping that carries elements of graph to numbers (usually to positive or non-negative integers). Here, the domain will be $V(G) \cup E(G)$. In other words, the labeling in this article is total labeling. In some labelings only the vertex set or the edge set will be used and we shall call them vertex-labeling or edge-labelings, respectively. Graph labelings has many types such as harmonius, radio, cordial, graceful and antimagic. The recent survey of graph labelings can be seen in [5]. In this paper, we will focus on antimagic total labeling type. In [1], more details on an antimagic total labeling can be seen. The notion of edge-magic total labeling of graphs derives its origin in the research work of A. Kotzig and A. Rosa [12, 13] for which they used the terminology magic valuation. Let us now move to few useful definitions and relevant research work previously done.

Definition 1. A (k, d) edge-antimagic vertex ($(k, d)-$EAV) labeling of a graph G is a bijection $\rho: V(G) \rightarrow \{1, 2, \cdots, p\}$ such that the set of edge-sums of all edges in G, $\{w(xy) = \rho(x) + \rho(y) : xy \in E(G)\}$, forms an arithmetic progression $\{k, k + d, k + 2d, \ldots, k + (q - 1)d\}$, where $k > 0$ and $d \geq 0$ are fixed integers.

R. Simanjuntak *et al*., [19] proved that cycles and path, C_{2n+1}, P_{2n+1} and P_{2n}, have an $(n + 2, 1)$-EAV labeling when $n \geq 1$. They further proved that the odd path P_{2n+1} has a $(n + 3, 1)$-EAV labeling and the path P_n admits a $(3, 2)$-EAV labeling for $n \geq 1$. In [3], M. Baca *et al*., proved that if a connected graph G (must not be a tree) has an (a, d)-EAV labeling then $d = 1$. Further that a cycle C_n has no (a, d)-EAV labeling for $d > 1$ and $n \geq 3$ [3].

Definition 2. A (k, d) edge-antimagic total labeling of a graph G is a bijection $\rho: V(G) \cup E(G) \rightarrow \{1, 2, \cdots, p + q\}$ such that the set of edge-weights of all edges in G, $\{w(xy) = \rho(x) + \rho(xy) + \rho(y) : xy \in E(G)\}$, forms an arithmetic progression $\{k, k+d, k+2d, \ldots, k+(q-1)d\}$, where $k > 0$ and $d \geq 0$ are fixed integers. The graph G, if admits such labeling, is called an (k, d) edge-antimagic total graph. (abbreviated as $(a, d) -$ EAT labeling/ graph)

Definition 3. An (k, d)-EAT labeling ρ is called a super (k, d) edge antimagic total labeling of G if $\rho(V(G)) \rightarrow \{1, 2, \cdots, v\}$. Thus, a super (k, d)- edge-antimagic total graph is a graph that admits a super (k, d) edge-antimagic total labeling. (abbreviated as super $(k, d) -$ EAT labeling/ graph). If $d = 0$, then a super $(k, 0)$-EAT labeling is called a super edge-magic total labeling and k is called a magic constant or valence. For d other than 0, k is called minimum edge-weight. These all (k, d) definitions related graphs are also termed as (k, d) arithmetic graphs.

The definition of an (*k*, *d*)- EAT labeling was established by R. Simanjuntak, Bertault and M. Miller in [19] as a natural extension of an edge-magic total labeling defined by A. Kotzig and A. Rosa earlier. A super (*k*, *d*)- EAT labeling is a further natural extension of the notion of a super (*k*, 0)-EAT labeling introduced by Hikoe Enomoto *et al.*, in [4]. And not to forget the following interesting conjecture of the same paper that every tree admits a super (*k*, 0) edge-antimagic total labeling. Many researchers have pillared this conjecture by deriving super (*k*, d)-EAT labeling for many particular classes of trees. As in, stars, path like trees, *W*-trees, subdivided stars, caterpillars and lobsters. All such results can be seen in [2, 6, 7, 9, 10, 11, 8, 15, 17, 18, 20]. And the famous computer calculated verification of upto 17 vertices tree [14].

Results

We are providing, apart from definitions of some dual connected graphs, their (*a*, 2) and hence (*a*, 0) edge- antimagic total labelling. Here the notion of dual connected graphs is different from that of the notion that one graph being the dual of another. The following result are our relevant calculations.

Theorem 1. *The dual connected graph G_n is (a, 2) edge-antimagic total, for n chosen to be even.*

Proof. We first define the notion of dual connected graph G_n with following vertex and edge sets,

$$V(G_n)=\{x_i, y_i : 1\le i\le n+4\}\cup\{c_i : 1\le i\le 16\}$$

$$E(G_n)=\{x_i\, x_{i+1} : 1\le i\le n+3\}\cup\{y_i\, y_{i+1} : 1\le i\le n+3\}\cup\{x_i\, y_{i+1} : 1\le i\le \frac{n-4}{2}\ \&\ \frac{n+12}{2}\le i\le n+3\}$$
$$\cup\{x_i\, y_{i+2} : 1\le i\le \frac{n-6}{2}\ \&\ \frac{n+12}{2}\le i\le n+2\}\cup\{x_i\, c_{2i-n+3}, y_i\, c_{2i-n+3} : \frac{n}{2}\le i\le \frac{n+10}{2}\}$$
$$\cup\{x_i\, c_{2i-n+4}, y_i\, c_{2i-n+4} : \frac{n}{2}\le i\le \frac{n+10}{2}\}\cup\{c_1\, y_2, c_2\, y_{\frac{n}{2}-2}, c_{15}\, x_{\frac{n}{2}+7}, c_{16}\, y_{n+4}\}$$
$$\cup\{c_1\, x_1, c_1\, y_1, c_{16}\, x_{n+4}, c_{16}\, y_{n+4}\}\cup\{c_2\, x_{\frac{n}{2}-1}, c_2\, y_{\frac{n}{2}-1}, c_{15}\, x_{\frac{n}{2}+6}, c_{15}\, x_{\frac{n}{2}+6}\}\cup\{c_i\, c_{i+1} : 2\le i\le 14\}.$$

Then $p=|V(G_n)|=2(n+12)$ and $q=|E(G_n)|=4(n+12)-3$. By defining a bijective function on G_n as $\delta : V(G_n)\to\{1,2,\dots,p\}$ as follows:

$$\delta(c_i)=\begin{cases}1 : i=1;\\ 2i+n-4 : 2\le i\le 14; i\equiv 0(\text{mod } 2);\\ 2i+n-5 : 3\le i\le 15; i\equiv 1(\text{mod } 2);\\ 2(n+12) : i=16\end{cases}$$

$$\delta(x_i)=\begin{cases}2i : 1\le i\le \frac{n-2}{2};\\ 4i-n+2 : \frac{n}{2}\le i\le \frac{n+12}{2};\\ 2(i+6)+2 : \frac{n+14}{2}\le i\le n+4\end{cases}$$

$$\delta(y_i)=\begin{cases}\delta(x_i)+1 : 1\le i\le \frac{n-2}{2};\\ \delta(x_i)+1 : \frac{n}{2}\le i\le \frac{n+12}{2};\\ \delta(x_i)+1 : \frac{n+14}{2}\le i\le n+4\end{cases}$$

($\delta(x_i)$ being chosen to be corresponding label in the respective range). It can be easily followed that with appropriate edge-labels δ refers to an (*a*, 2) and hence (*a*, 0) edge-antimagic total labeling of G_n, which are assigned in same and in opposite order respectively.

Theorem 2. *The dual connected graph H_n nonisomorphic to G_n is (a, 2) edge- antimagic total, for n choosen to be even.*

Proof. We define the notion of dual connected graph H_n with following vertex and edge sets,

$$V(H_n)=\{x_i, y_i : 1\le i\le n+4\}\cup\{c_i : 1\le i\le 16\}$$

$$E(H_n)=\{x_i\,x_{i+1}:1\le i\le n+3\}\cup\{y_i\,y_{i+1}:1\le i\le n+3\}\cup\{x_i\,y_{i+1}:1\le i\le \frac{n-4}{2}\ \&\ \frac{n+12}{2}\le i\le n+3\}$$
$$\cup\{x_i\,y_{i+2}:1\le i\le \frac{n-6}{2}\ \&\ \frac{n+12}{2}\le i\le n+2\}\cup\{x_i\,c_{2i-n+3},y_i\,c_{2i-n+3}:\frac{n}{2}\le i\le \frac{n+10}{2}\}$$
$$\cup\{x_i\,c_{2i-n+4},y_i\,c_{2i-n+4}:\frac{n}{2}\le i\le \frac{n+10}{2}\}\cup\{c_1\,y_2,c_2\,y_{\frac{n}{2}-2},c_{15}\,x_{\frac{n}{2}+7},c_{16}\,y_{n+4}\}$$
$$\cup\{c_1\,x_1,c_1\,y_1,c_{16}x_{n+4},c_{16}\,y_{n+4}\}\cup\{c_2x_{\frac{n}{2}-1},c_2\,y_{\frac{n}{2}-1},c_{15}x_{\frac{n}{2}+6},c_{15}\,y_{\frac{n}{2}+6}\}\cup\{c_i\,c_{i+1}:2\le i\le 14,i\equiv 0(\mathrm{mod}\ 2)\}$$
$$\cup\{x_iy_i:\frac{n}{2}\le i\le \frac{n+10}{2}\}.$$

Then $p=|V(H_n)|=2(n+12)$ and $q=|E(H_n)|=4(n+12)-3$. By defining a bijective function on H_n as $\varphi:V(H_n)\to\{1,2,...,p\}$ as follows:

$$\varphi(c_i)=\begin{cases}1:i=1;\\ 2i+n-4:2\le i\le 14;i\equiv 0(\mathrm{mod}\ 2);\\ 2i+n-5:3\le i\le 15;i\equiv 1(\mathrm{mod}\ 2);\\ 2(n+12):i=16\end{cases}$$

$$\varphi(y_i)=\begin{cases}\varphi(x_i)+1:1\le i\le \frac{n-2}{2};\\ \varphi(x_i)+1:\frac{n}{2}\le i\le \frac{n+12}{2};\\ \varphi(x_i)+1:\frac{n+14}{2}\le i\le n+4\end{cases}$$

($\varphi(x_i)$ being choosen to be corresponding label in the respective range). It can be followed that with appropriate edge-labels φ refers to an (a, 2) and hence (a, 0) edge-antimagic total labeling of G_n, which are once again assigned in same and in opposite order respectively.

Theorem 3. *The dual connected graph G_n is (a, 2) edge-antimagic total, for n choosen to be odd.*

Proof. We define the dual connected graph G_n with following vertex and edge sets, for even n as;

$$V(G_n)=\{x_i,y_i:1\le i\le n+4\}\cup\{c_i:1\le i\le 16\}$$

$$\varphi(x_i)=\begin{cases}2i:1\le i\le \frac{n-2}{2};\\ 4i-n+2:\frac{n}{2}\le i\le \frac{n+12}{2};\\ 2(i+6)+2:\frac{n+14}{2}\le i\le n+4\end{cases}$$

$$E(G_n)=\{x_i x_{i+1}:1\le i\le n+4\}\cup\{y_i y_{i+1}:1\le i\le n+4\}\cup\{y_i x_{i+1}:1\le i\le \frac{n-3}{2} \,\&\, \frac{n+13}{2}\le i\le n+4\}$$
$$\cup\{x_i y_{i+2}:1\le i\le \frac{n-5}{2} \,\&\, \frac{n+13}{2}\le i\le n+2\}\cup\{x_i c_{2i-n+1}, y_i c_{2i-n+1}:\frac{n+1}{2}\le i\le\frac{n+11}{2}\}$$
$$\cup\{x_i c_{2i-n+2}, y_i c_{2i-n+2}:\frac{n+1}{2}\le i\le\frac{n+11}{2}\}\cup\{c_1 x_{\frac{n-1}{2}}, c_1 y_{\frac{n-1}{2}}, c_{14} x_{\frac{n+13}{2}}, c_{14} y_{\frac{n+13}{2}}\}$$
$$\cup\{c_1 y_1, x_{n+5} y_{n+5}, c_5 x_{\frac{n-3}{2}}, c_{14} y_{\frac{n+15}{2}}\}\cup\{c_i c_{i+1}:1\le i\le 13\}.$$

Then $p=|V(G_n)|=2(n+12)$ and $q=|E(G_n)|=4(n+12)-3$. By defining a bijective function on G_n as $\gamma:V(G_n)\to\{1,2,\ldots,p\}$ as follows:

$$\gamma(c_i)=\begin{cases}2i+n-2: 2\le i\le 13; i\equiv 1(\text{mod } 2);\\ 2i+n-3: 2\le i\le 14; i\equiv 0(\text{mod } 2);\end{cases}$$

$$\gamma(x_i)=\begin{cases}2i: 1\le i\le\frac{n-1}{2};\\ 4i-n+1:\frac{n+1}{2}\le i\le\frac{n+13}{2};\\ 2(i+6)+2:\frac{n+15}{2}\le i\le n+4\end{cases}$$

$$\gamma(y_i)=\begin{cases}\gamma(x_i)-1: 1\le i\le\frac{n-1}{2};\\ \gamma(x_i)-1:\frac{n+1}{2}\le i\le\frac{n+13}{2};\\ \gamma(x_i)-1:\frac{n+15}{2}\le i\le n+4\end{cases}$$

($\gamma(x_i)$ being chosen to be corresponding labels). It follows with appropriate edge- labels γ refers to an (*a*, 2) and hence (*a*, 0) edge-antimagic total labeling of G_n, which are assigned in same and in opposite order respectively.

Theorem 4. *The dual connected graph H_n nonisomorphic to G_n (defined in Theorem 3) is (a, 2) edge- antimagic total, for n choosen to be even.*

Proof. We define the notion of dual connected graph H_n with following vertex, edge connection,

$$V(H_n)=\{x_i, y_i:1\le i\le n+4\}\cup\{c_i:1\le i\le 16\}$$

$$E(H_n)=\{x_i x_{i+1}:1\le i\le n+4\}\cup\{y_i y_{i+1}:1\le i\le n+4\}\cup\{y_i x_{i+1}:1\le i\le \frac{n-3}{2} \,\&\, \frac{n+13}{2}\le i\le n+4\}$$
$$\cup\{x_i y_{i+2}:1\le i\le \frac{n-5}{2} \,\&\, \frac{n+13}{2}\le i\le n+2\}\cup\{x_i c_{2i-n+1}, y_i c_{2i-n+1}:\frac{n+1}{2}\le i\le\frac{n+11}{2}\}$$
$$\cup\{x_i c_{2i-n+2}, y_i c_{2i-n+2}:\frac{n+1}{2}\le i\le\frac{n+11}{2}\}\cup\{c_1 x_{\frac{n-1}{2}}, c_1 y_{\frac{n-1}{2}}, c_{14} x_{\frac{n+13}{2}}, c_{14} y_{\frac{n+13}{2}}\}$$
$$\cup\{x_1 y_1, x_{n+5} y_{n+5}, c_1 x_{\frac{n-3}{2}}, c_{14} y_{\frac{n+15}{2}}\}\cup\{c_i c_{i+1}:1\le i\le 13; i\equiv 1(\text{mod } 2)\}\cup\{x_i y_i:\frac{n+1}{2}\le i\le\frac{n+11}{2}\}.$$

Then $p=|V(G_n)|=2(n+12)$ and $q=|E(G_n)|=4(n+12)-3$. By defining a bijective function on G_n as $\lambda:V(G_n)\to\{1,2,\ldots,2(n+12)\}$ as follows:

$$\lambda(c_i)=\begin{cases}\gamma(c_i): 2\le i\le 13; i\equiv 1(\text{mod } 2);\\ \gamma(c_i): 2\le i\le 14; i\equiv 0(\text{mod } 2);\end{cases}$$

$$\lambda(x_i) = \begin{cases} \gamma(x_i) : 1 \le i \le \frac{n-1}{2}; \\ \gamma(x_i) : \frac{n+1}{2} \le i \le \frac{n+13}{2}; \\ \gamma(x_i) : \frac{n+15}{2} \le i \le n+4 \end{cases}$$

$$\lambda(y_i) = \begin{cases} \lambda(x_i) - 1 : 1 \le i \le \frac{n-1}{2}; \\ \lambda(x_i) - 1 : \frac{n+1}{2} \le i \le \frac{n+13}{2}; \\ \lambda(x_i) - 1 : \frac{n+15}{2} \le i \le n+4 \end{cases}$$

(λ and γ being choosen to be corresponding labels). It can be easily followed that with appropriate edge- labels λ refers to an (a, 2) and hence (a, 0) edge-antimagic total labeling of H_n, which are assigned in same and in opposite order respectively.

Conclusion

We have mainly provided super (a, 0) and (a, 2) edge- antimagic labeling of dual connected graphs G_n and H_n. Wherein, an (a, d) edge-antimagic total labeling of a graph G is a bijection $\rho: V(G) \cup E(G) \rightarrow \{1, 2, \ldots, p+q\}$ such that the set of edge-weights of all edges in G, $\{w(xy) = \rho(x) + \rho(xy) + \rho(y) : xy \in E(G)\}$, forms an arithmetic progression $\{a, a+d, a+2d, \ldots, a+(q-1)d\}$, where $a > 0$ and $d \ge 0$ are fixed integers, Our main focus here is $a = 0$ and $a = 2$.

Acknowledgement

I am deeply thankful to the referee for valuable suggestions which have improved the final version of this paper.

References

[1] M. Baˇca and M. Miller, Super Edge-Antimagic Graphs, Brown Walker Press, Boca Raton, Florida USA, 2008.

[2] M. Baˇca, Y. Lin and F. A. Muntaner-Batle, Super edge-antimagic labelings of the path-like trees, Util. Math. 73 (2007) 117-128.

[3] M. Baˇca, Y. Lin, M. Miller and R. Simanjuntak, New constructions of magic and antimagic graph labelings, Util. Math. 60 (2001) 229-239.

[4] H. Enomoto, A. S. Llado, T. Nakamigawa and G. Ringel, Super edge-magic graphs, SUT J. Math. 34 (1998) 105-109.

[5] J. A. Gallian, A dynamic survey of graph labeling, Electron. J. Combin, 2017.

[6] M. Hussain, E. T. Baskoro and Slamin, On super edge-magic total labeling of banana trees, Util. Math. 79 (2009) 243-251.

[7] M. Javaid, M. Hussain, K. Ali and H. Shaker, On super edge-magic total labeling on subdivision of trees, Util. Math. 89 (2012) 169-177.

[8] M. Javaid, A. A. Bhatti, M. Hussain and K. Ali, Super edge-magic total labeling on forest of extended W-trees, Util. Math. 91 (2013) 155-162.

[9] M. Javaid and A. A. Bhatti, On super (*a, d*)-edge antimagic total labeling of subdivided stars, Ars Combin. 105 (2012) 503-512.

[10] M. Javaid, A. A. Bhatti and M. Hussain, On (*a, d*)-edge-antimagic total labeling of extended *W*-trees, Util. Math. 87 (2012) 293-303.

[11] M. Javaid, M. Hussain, K. Ali and K. H. Dar, Super edge-magic total labeling on *W*-trees, Util. Math. 86 (2011) 183-191.

[12] A. Kotzig and A. Rosa, Magic valuations of finite graphs, Canad. Math. Bull., 13 (1970) 451-461.

[13] A. Kotzig and A. Rosa, Magic Valuation of Complete Graphs, Centre de Recherches Mathematiques, Uni. de Montreal, 1972.

[14] S. M. Lee and Q. X. Shah, All trees with at most 17 vertices are super edge-magic, 16th MCCCC Conference, Carbondale SIU (2002).

[15] Y. J. Lu, A proof of three-path trees $P_{(m,\ n,\ t)}$ being edge-magic (II),College Math- ematica, 20(3) (2004) 51-53.

[16] A. M. Marr, W. D. Wallis, Magic Graphs, Springer New York Heidelberg Dordrecht London, 2013.

[17] A. A. G. Ngurah, R. Simanjuntak and E. T. Baskoro, On super edge-magic total labeling of subdivision of $K_{1,3}$ SUT J. Math. 43 (2007) 127-136.

[18] A. N. M. Salman, A. A. G. Ngurah and N. Izzati, On super edge-magic total labeling of a subdivision of a star S_n, Util. Math. 81 (2010) 275-284.

[19] R. Simanjuntak, F. Bertault and M. Miller, Two new (*a, d*)-antimagic graph labelings, 11th Australian Workshop on Combin. Algor., 11 (2000) 179-189.

[20] K. A. Sugeng, M. Miller, Slamin and M. Baca, (*a, d*) edge-antimagic total labelings of caterpillars, Lect. Notes Comput. Sci. 3330 (2005) 169-180.

[21] D. B. West, An Introduction to Graph Theory, Prentice Hall, 1996.