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Full Length Research Paper

Relativistic Wave Equation for a Stretched String

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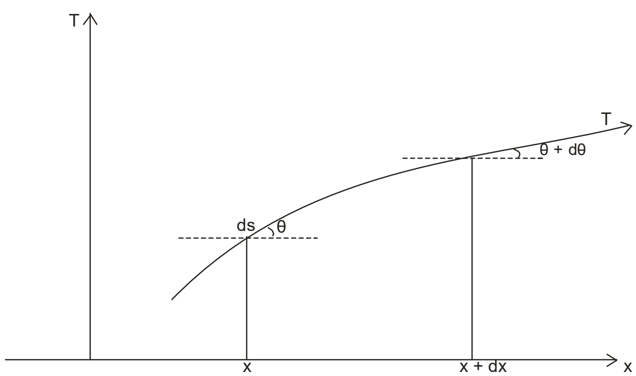
ABSTRACT

The theory of Newtonian mechanics for the description of the particle behavior of all particles of non-zero rest masses in all interactive fields in nature is limited to massive particles that move with a speed not closer to the speed of light. For particles that move at the speed closer or at the speed of light, Newtonian mechanics could not hold. In this paper we are out to show how to derive the wave equation for a stretched string based upon Einstein's theory of classical mechanics called Special Relativity as an extension to the well-known wave equation for a stretched string in classical mechanics for applications in sub-atomic particles.

1.0 Consider a uniform string under a constant tension, T. Let y be the instantaneous vertical displacement of an element of the string located at the horizontal position x at the time t, as shown in figure1



Let λ_0 be the uniform linear density of rest mass of the string. Then neglecting the effect of gravitation then, the vertical force acting at the end of the element at x is given by $Tsin\theta$. Also the vertical force acting at the end of the string at x+dx is giving by $Tsin(\theta + d\theta)$. consequently, the net vertical force acting on the element of the string, F, is given by;

$$F = T[\sin(\theta + d\theta) - sin\theta] \quad (1)$$

But under the assumption of small θ it follows from figure 1 and trigonometry that:

$$\sin(\theta + d\theta) - sin\theta \approx y_{xx} dx \quad (2)$$

Where

$$y_x = \frac{\partial y}{\partial x} \quad (3)$$

Thus the force (1) acting on the element becomes

$$F \approx T y_{xx} dx \quad (4)$$

Next the inertial mass of the element of the string, dm_I, is given by

$$dm_I = \lambda_0 ds \qquad (5)$$

where ds is an element of an arc along the string given by

$$ds = (1 + y_x^2)^{\frac{1}{2}} dx \qquad (6)$$

It therefore follows from Newton's laws and (4), (5) and (6) that the equation of motion of the element is given by

$$\frac{\partial}{\partial t}\left\{(1 + y_x^2)^{\frac{1}{2}} y_t\right\} - \frac{1}{v^2} y_{xx} = 0 \qquad (7)$$

Where;

$$y_t = \frac{\partial y}{\partial t} \qquad (8)$$

And

$$v = \left(\frac{T}{\lambda_0}\right)^{-\frac{1}{2}} \qquad (9)$$

This is Newton's equation of motion for the string.
In the limit of small oscillations,

$$1 \gg y_x \qquad (10)$$

Newton's equation of motion for the string reduces to

$$y_{tt} - \frac{1}{v^2} y_{xx} = 0 \qquad (11)$$

This is the well-known homogeneous wave equation [1-2] with propagation speed v. We shall next derive the corresponding wave equation for the string based upon Einstein's Theory of Special Relativity.

2.0 Mathematical Analysis

According to Einstein's Theory of Special Relativity the inertial mass m_I of a particle of nonzero rest mass m_0 is given [3-22] by

$$m_1 = \left(1 - \frac{u^2}{v^2}\right)^{-\frac{1}{2}} m_0 \qquad (12)$$

where u is the instantaneous speed and c is the speed of light in vacuo. Thus for the string in figure 1, the element of inertial mass dm_I is given by

$$dm_I = \lambda_0 \left(1 - \frac{y_t^2}{c^2}\right)^{-\frac{1}{2}} ds \qquad (13)$$

Consequently, the equation of motion for the element according to Einstein's Theory of Special Relativity follows from (13), (4) and (6) as

$$\frac{\partial}{\partial t}\left\{(1 + y_x^2)^{\frac{1}{2}} \left(1 - \frac{y_t^2}{c^2}\right)^{-\frac{1}{2}} y_t\right\} - \frac{1}{v^2} y_{xx} = 0 \qquad (14)$$

Where; v is given by (9). This is the equation of motion for the string according to the Einstein's Theory of Special Relativity.

To the order of c^0 the relativistic equation of motion for the string (14) reduces to the pure Newtonian equation of motion (7)

To the order of c^{-2} the relativistic equation of motion for the string (14) reduces to;

$$\frac{\partial}{\partial t}\left\{(1 + y_t^2)^{\frac{1}{2}} y_t\right\} + \frac{1}{2c^2}\frac{\partial}{\partial t}\left\{(1 + y_x^2)^{\frac{1}{2}} y_t^3\right\} - \frac{1}{v^2} y_{xx} = 0 \qquad (15)$$

which shows clearly the first Post-Newtonian correction term.

In the limit of small oscillation (10) the relativistic equation of motion for the string (14) reduces to;

$$\frac{\partial}{\partial t}\left\{\left(1 - \frac{y_t^2}{c^2}\right)^{-\frac{1}{2}} y_t\right\} - \frac{1}{v^2} y_{xx} = 0 \qquad (16)$$

This is the relativistic analog of Newton's wave equation for the string (11). To the order of $c^0$the relativistic wave equation for the string (16) reduces to the Pure Newtonian wave Equation (11). To the order of c^{-2} the relativistic wave equation for the string (16) reduces to;

$$y_{tt} + \frac{3}{2} y_t^2 y_{tt} - \frac{1}{v^2} y_{xx} = 0 \qquad (17)$$

which shows clearly the first Post-Newtonian correction term.

3:0 Summary and Conclusion

In this paper we derived the Special Relativistic Generalization of the Equation of Motion of a stretched string (14) and the corresponding "wave'' equation (16). These two new equations are hence forth uncovered for mathematical analysis and integration, and subsequent physical interpretation and experimental investigation and applications.

Finally, this paper opens the way for the derivation of the Special Relativistic Generalizations of all classical wave equations such as those for membranes and pipes.

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