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Research Paper

On the Generation of Nuclear Data Covariances in the Context of Efficiency Calibration of Ge(Li) Detectors

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ABSTRACT

This paper describes procedure to generate covariance matrix in the context of efficiency calibration of Ge(Li) detectors using partial uncertainties and micro-correlations. In order to keep the article self contained a) Complete raw data is presented, b) Procedure to generate covariance matrix using partial uncertainties and micro-correlations is outlined in the context of efficiency calibration of Ge(Li) detector. Note that the raw data presented in this article is extracted from the published articles [1, 2, 3], for demonstration purpose. The procedure outlined here is applicable to the nuclear data available in EXFOR library in the process of nuclear data evaluations.

1. Introduction

The numerical data extracted from original publications related to experimental nuclear physics is inadequate for nuclear physics applications; instead they rely on evaluated (or recommended) nuclear data (nuclear reaction and nuclear structure data). The evaluated nuclear data is generated by experienced nuclear data evaluators, by carefully combining (using advanced statistical methods) numerical results from both experiments and theoretical models. This paper focus on the data reduction and data evaluation in the context of efficiency calibration of HPGe or Ge(Li) detector. The methodology presented in this paper is applicable in the evaluation of nuclear reaction data (e.g., neutron induced reaction cross-sections, compiled in EXFOR database) and also useful in the analysis of experimental data in general.

1.1 Experimental context and data

Suppose an experimenter perform n distinct measurements to determine absolute full energy peak efficiency (ϵi) of the HPGe or Ge(Li) detector corresponding to gamma energies (Ei). The aim is to generate pair (ϵi, Ei) of calibration points at a source to detector distance of approximately 20 cm. The experiment was conducted in March 1988, as a part of neutron activation experiment. The point sources used for calibration were [2]: ^{60}Co (5.271(0.001) y) with an activity 3.193×105 (±0.9%) *Bq.* was recorded in February 1984, ^{137}Cs (30.174(0.034) y) with an activity 1.565×105 (±1.5%) *Bq.* was recorded in January 1985 and ^{152}Eu (13.33(0.04) y) with an activity 4.208×105 (±1.5%) *Bq.* was recorded in April 1979. The gamma counts per live time (full energy peak yields) C for each of 12 lines, gamma energies and abundances are presented in Table 1. The counting experiment is repeated n times and the count data presented are the weighted averages.

2. Errors, uncertainties and covariance matrix

The gamma counts C and efficiency of the detector ϵ are related through the equation

$$C = Aa\epsilon, \quad (1)$$

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Table 1. Gamma lines, gamma abundances and gamma counts[1].

Line No.	Source	Eγ (keV)	Iγ (%)	C
1	^{60}Co	1173	100(0.01)	56.64±0.23
2		1333	100(0.01)	51.93±0.21
3	^{137}Cs	662	85.6(0.1)	62.04±0.19
4	^{152}Eu	245	7.42(2.1)	24.99±0.05
5		344	26.4(1.5)	65.02±0.07
6		444	3.08(1.6)	6.00±0.02
7		779	13.0(1.7)	14.89±0.04
8		867	4.16(1.3)	4.45±0.02
9		964	14.5(1.6)	14.17±0.03
10		1086	11.8(2.2)	10.40±0.03
11		1112	13.6(1.5)	11.86±0.02
12		1408	20.7(1.3)	14.74±0.03

Where; a is gamma abundance and activity A at the time of measurement is related to activity A0 and decay constant (λ) by decay law

$$A = A_0 e^{-\lambda t}. \quad (2)$$

All the required information to determine ϵ at 12 calibration points are presented in Section. I. Note that data are presented in the form ⟨x⟩±Δx, because true value of a physical quantity xtrue is given by xtrue = ⟨x⟩+δ_x, where ⟨x⟩ is the best estimate of x and δ_x is the associated error. The error and true value, both remain unknown, but our knowledge about the error can be represented by defining uncertainty $\Delta_x = \sqrt{(\delta_x)2}$, which can be estimated from the experiment. In the context of efficiency calibration, there are four distinct component errors δC, δa, δλ and δA_0. In linear approximation total error δϵ is related to component errors (total error is approximately equal to sum of partial errors) as

$$\delta\epsilon \approx \frac{\partial\epsilon}{\partial C}\delta C + \frac{\partial\epsilon}{\partial a}\delta a + \frac{\partial\epsilon}{\partial\lambda}\delta\lambda + \frac{\partial\epsilon}{\partial A_0}\delta A_0. \quad (3)$$

By using Eq.(1), Eq.(2) in Eq.(3) and after simplification total uncertainty $\Delta\epsilon = \sqrt{\langle(\delta\epsilon)2\rangle}$ is given by

$$\frac{\Delta\epsilon}{\langle\epsilon\rangle} = \sqrt{(\frac{\Delta C}{\langle C\rangle})^2 + (\frac{\Delta a}{\langle a\rangle})^2 + (\frac{\Delta\lambda}{\langle\lambda\rangle})^2 + (\frac{\Delta A_0}{\langle A_0\rangle})^2}, \quad (4)$$

where ⟨ϵ⟩ is the best estimate of ϵ obtained in accordance with ⟨C⟩ = ⟨A⟩⟨a⟩⟨ϵ⟩. Recast Eq.(4) as

$$\Delta\epsilon = \sqrt{(\frac{\Delta C}{\langle C\rangle}\langle\epsilon\rangle)^2 + (\frac{\Delta a}{\langle a\rangle}\langle\epsilon\rangle)^2 + (\frac{\Delta\lambda}{\langle\lambda\rangle}\langle\epsilon\rangle)^2 + (\frac{\Delta A_0}{\langle A_0\rangle}\langle\epsilon\rangle)^2}, \quad (5)$$

$$\Delta\epsilon = \sqrt{e_1^2 + e_2^2 + e_3^2 + e_4^2}, \quad (6)$$

where, e1, e2, e3, and e4 are partial uncertainties[3] having the dimension of ϵ. i.e., square of total uncertainty (variance) is just sume of square of partial uncertainties.

Similarly, the covariance between two errors Δϵij = $\sqrt{\langle\delta\epsilon i\delta\epsilon j\rangle}$ is given by

$$V_{ij} \equiv \Delta\epsilon_{ij} = S_{lij}e_{li}e_{lj}, \quad (7)$$

where *l* = 1, 2, 3, 4 corresponds to attributes (or different components), e.g.,

$$e_{1i} = \frac{\Delta C_i}{\langle C_i\rangle}\langle\epsilon_i\rangle.$$

S corresponds to micro-correlation, e.g., S1ij is the correlation within the first attribute. If errors are categorized into random and systematic errors, it will be easier to assign micro-correlations. Among the four component errors, except δC, remaining three component errors are known prior to the counting measurement and are adopted from different sources. For e.g, Activity, decay constant and gamma abundances are either measured by someone else or they can be even evaluated values. We don't have freedom to minimize the errors, where as just by increasing the counting measurements δC can be reduced. Those errors which are directly connected with the current measurement and which can be reduced by increasing number of experiments, such errors are called random errors and all other error component errors are called systematic errors. In the context of efficiency calibration only δCi are random errors, remaining error components are

systematic. Micro-correlations between random errors are assigned zero, where as micro-correlations between systematic errors are assigned one if a component systematic error is common to whole data base, else it is zero. If there is additional information is available, then only micro-correlations between a component systematic errors can be assigned values other than zero and one. Note that in Eq.(7) if i = j reduces to square of Δϵi (Eq.(6)). i.e., diagonal entries in covariance matrix Vij corresponds to variances (Δϵi)2 and off diagonal entries correspond to covariances Δϵij. Hence, it is customary to publish covariance matrix along with mean values in the research articles pertained to experimental nuclear physics.

3. Result and discussions

Using the formalism outlined in Section. 2 and data presented in Section. 1, first generate table of partial uncertainties as presented in Table 2 .

Table 2. Table of partial uncertainties ($\times 10^6$)[3].

$l = 1$	$l = 2$	$l = 3$	$l = 4$
1.2356	0	0.058604	2.7801
1.1132	0	0.052798	2.5047
1.5048	3.0096	0.565202	7.5240
2.5380	26.6490	3.807952	19.0350
0.9278	13.9170	2.784096	13.9170
2.9348	11.7392	2.201650	11.0055
1.2945	7.3355	1.294824	6.4725
1.6124	5.2403	1.209602	6.0465
0.7362	5.8896	1.104576	5.5215
0.9960	7.3040	0.996249	4.9800
0.6568	4.9260	0.985446	4.9260
0.5366	3.4879	0.805101	4.0245

In order to save space micro-correlation matrices are not presented here, micro-correlation matrices for attribute l = 1 and l = 2 are identity matrices of order 12 × 12, whereas micro-correlation matrices for attribute 3 and 4 are not simple identity matrices. Interested readers are referred to [3] to learn more about micro-correlation matrices. From the table of partial uncertainties, generate 12× 12 diagonal matrices of partial uncertainties. A matrix computation represented by Eq.(7) generates 12×12 covariance matrices, square root of diagonal entries represents uncertainties in efficiencies. The result is presented in table 3.

4. Conclusions

The analysis presented in this paper on the generation of covariances in the context of efficiency calibration is also applicable in the experiments to determine reaction cross-sections. In the published articles and EXFOR database required information for covariance analysis and data evaluation is missing in maximum cases. The method presented in this paper will be applied to selected data files of EXFOR in future.

Table 3. Calculated efficiencies and their uncertainties expressed in percentage.

Line No.	$\{\epsilon\} \times 10^4$	$\Delta\epsilon / \{\epsilon\} \times 100$
1	3.089	1.0
2	2.783	1.0
3	5.016	1.6
4	12.69	2.6
5	9.278	2.1
6	7.337	2.2
7	4.315	2.3
8	4.031	2.0
9	3.681	2.2
10	3.320	2.7
11	3.284	2.1
12	2.683	2.0

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